A Particle in Motion

Agenda today

1. The relationship between velocity, acceleration and displacement
2. Projectile motion
3. Circular movement
4. Relative movement

Ockham’s razor and modeling

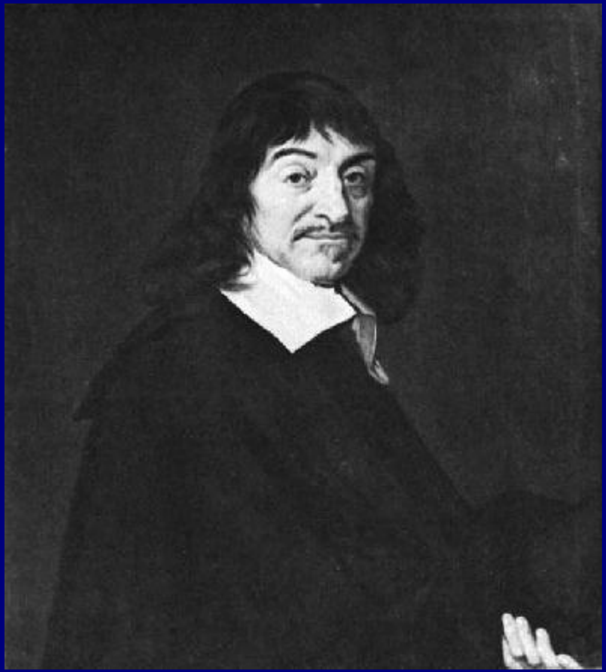
Particle（质点）

A point with no shape and size ,but has mass

Translational movement（平动）

Each point of the body has the same trail in the motion

Reference Frames（参照系）



"I am thinking therefore I exist."
(Latin: Cogito ergo sum)
from the *Discourse on Method*

Coordinate System：（坐标系）

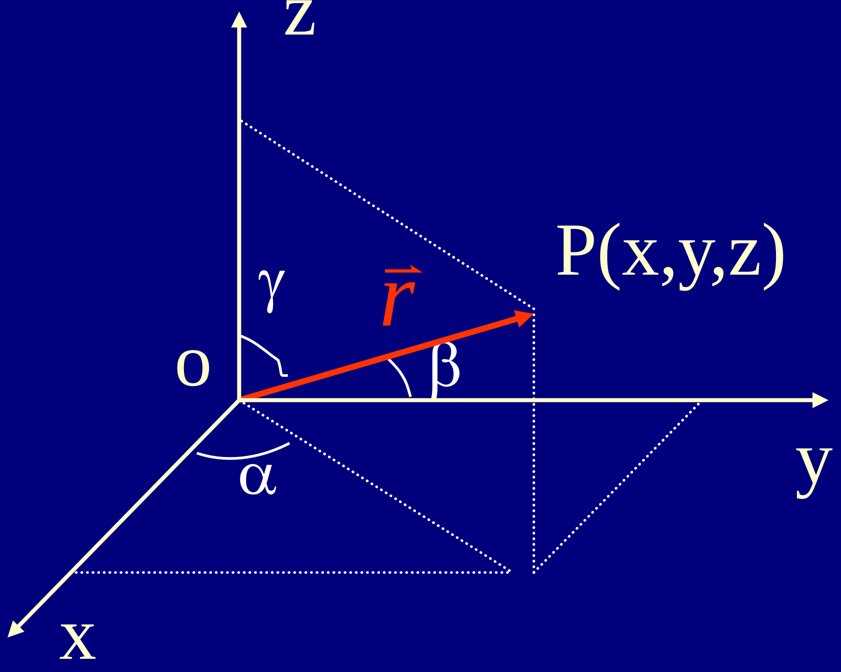
Cartesian coordinate system（直角坐标系）

polar system（极坐标系）

Position and Displacement Vector(位置矢量 和位移矢量）

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$



$$\cos\alpha = \frac{X}{r} \quad \cos\beta = \frac{y}{r} \quad \cos\gamma = \frac{z}{r}$$

$$\vec{r} = \vec{r}(t)$$

In Cartesian coordinate system

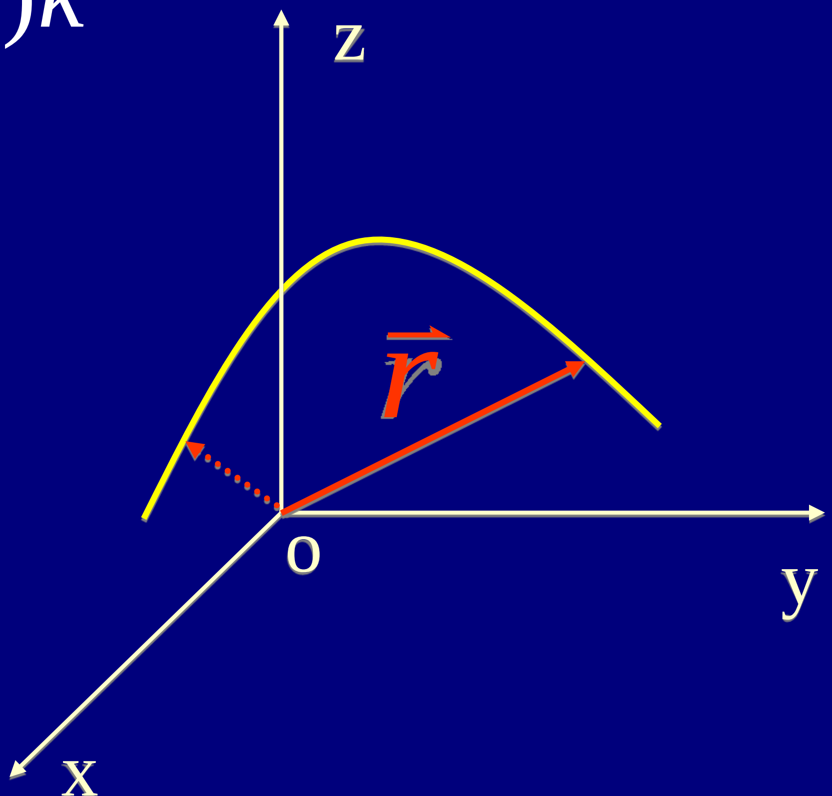
$$\vec{r} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

In component form

$$x = x(t)$$

$$y = y(t)$$

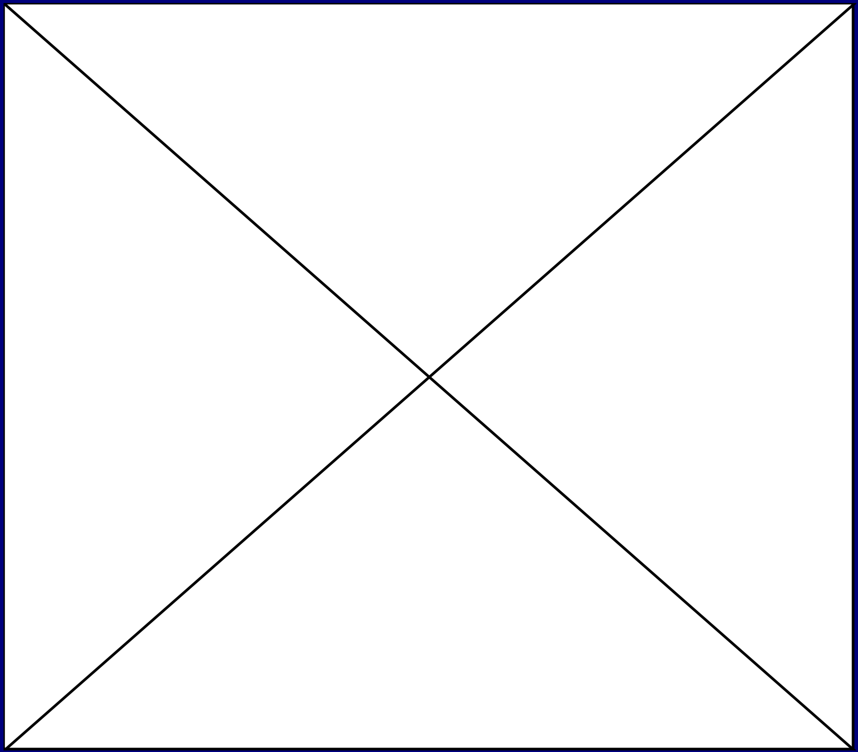
$$z = z(t)$$



$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A = \overrightarrow{AB}$$

$$\Delta\vec{r} = \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k}$$

$$\left|\Delta\vec{r}\right| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$



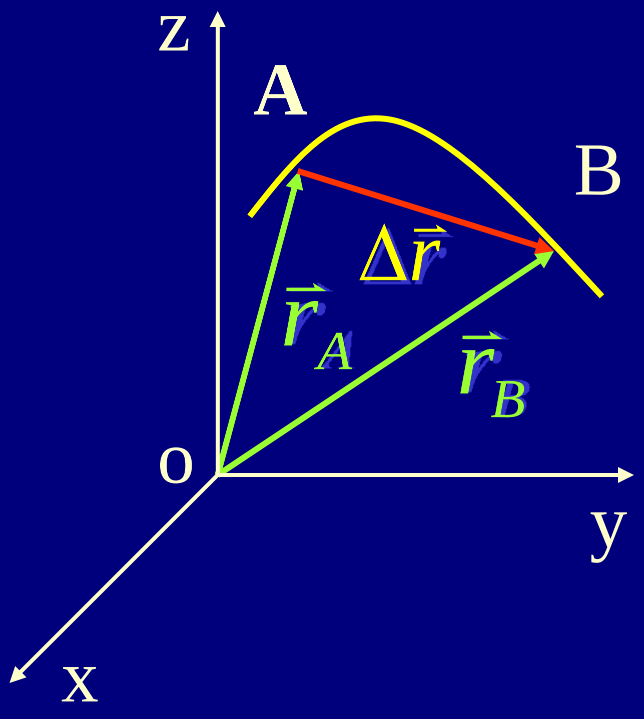
Displacement Vector

Velocity and Speed

Velocity describe the rate of change of the position of the particle on its trajectory.

Average Velocity:（平均速度）

$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} \qquad (m/s)$$



Instantaneous Velocity（瞬时速度）：

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad \left(m \cdot s^{-1}\right)$$

$$\vec{v} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k}$$

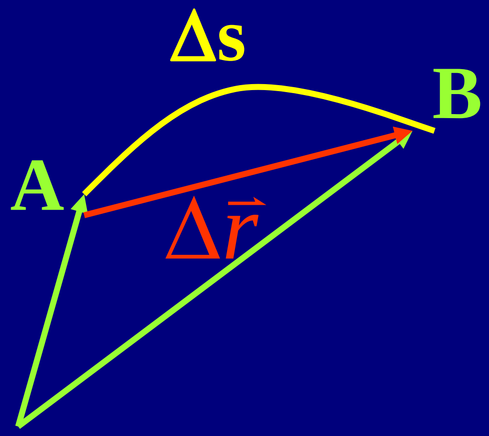
$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Speed（速率）

Average speed（平均速率）

$$\bar{v} = \frac{\Delta s}{\Delta t} \qquad \left(m \cdot s^{-1}\right)$$



Instantaneous speed（即时速率）

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Instantaneous Acceleration:

瞬时加速度：

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 r}{dt^2} \qquad \left(m \cdot s^{-2}\right)$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2} \qquad a_y = \frac{dv_y}{dt} = \frac{d^2 y}{dt^2} \qquad a_z = \frac{dv_z}{dt} = \frac{d^2 z}{dt^2}$$

$$\left|\vec{a}\right| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Can Achilles overtake the tortoise ?

Motion with non-constant Acceleration

$$\vec{r} = \vec{r}(t) \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$d\vec{v} = \vec{a}dt \quad , \quad \int_{\vec{v}_0}^{\vec{v}} d\vec{v} = \int_{t_0}^{t} \vec{a}dt$$

$$d\vec{r} = \vec{v}dt \quad , \quad \int_{\vec{r}_0}^{\vec{r}} d\vec{r} = \int_{t_0}^{t} \vec{v}dt$$

Projectile motion

The Horizontal motion

$$x - x_0 = (v\cos\theta) * t$$

The vertical motion

$$y - y_0 = (v\sin\theta) * t - \frac{1}{2}gt^2$$

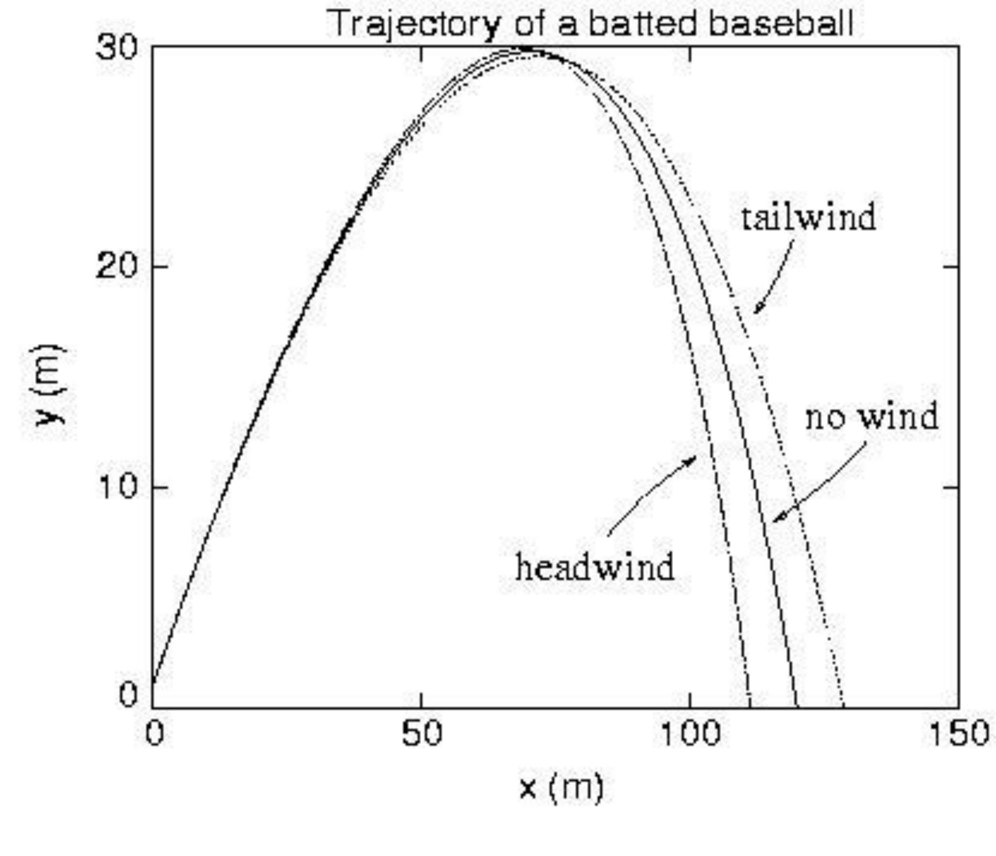
The equation of the Path

Let $x_0=0$, $y_0=0$

$$y = (\tan\theta)x - \frac{gx^2}{2(v_0\cos\theta)^2}$$

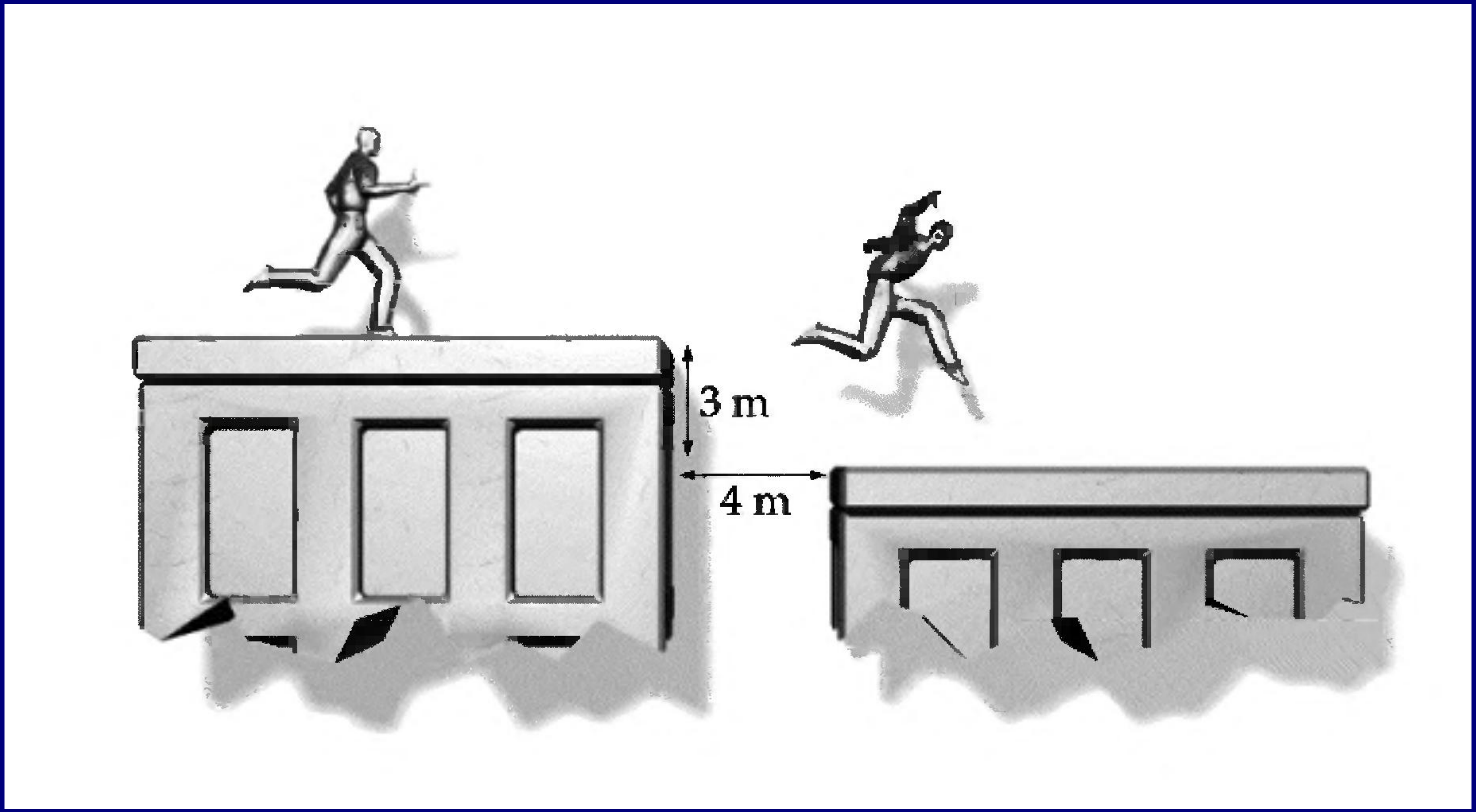
The Horizontal range

$$R = \frac{v_0^2}{g}\sin 2\theta$$



The effects of the air

- A policeman chases a master jewel thief across city rooftops. They are both running at 5 m/s when they come to a gap between buildings that is 4 m wide and has a drop of 3 m .The thief, having studied a little physics, leaps at 5 m/s and at 45° and clears the gap easily. The policeman did not study physics and thinks he should maximize his horizontal velocity, so he leaps at 5 m/s horizontally. (a) Does he clear the gap? (b) By how much does the thief clear the gap?


3 m
4 m

Example: The position of a certain particle is given by:

$$\vec{r} = 2t\vec{i} + \left(19 - 2t^2\right)\vec{j}$$

Find：（1）the trajectory of the particle；（2）position , velocity, acceleration of the particle when t=2；（3）when is the velocity perpendicular to the position vector？

Solution: **(1)**

$$x = 2t \quad , \quad y = 19 - 2t^2$$

$$y = 19 - \frac{1}{2}x^2$$

（2）

$$\vec{r}\big|_{t=2} = 2\times 2\vec{i} + \left(19 - 2\times 2^2\right)\vec{j} = 4\vec{i} + 11\vec{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\vec{i} - 4t\vec{j}$$

$$\vec{v}\big|_{t=2} = 2\vec{i} - 8\vec{j} \qquad \left(m\cdot s^{-1}\right)$$

$$v_{t=2} = \sqrt{2^2 + (-8)^2} = 8.25 m\cdot s^{-1}$$

$$\alpha = tg^{-1}\frac{-8}{2} = -75°58'$$

（3） $\vec{v} = \frac{d\vec{r}}{dt} = 2\vec{i} - 4t\vec{j}$ $\vec{a} = \frac{d\vec{v}}{dt} = -4\vec{j}$

$$\vec{a} = -4\vec{j}(m \cdot s^{-1})$$

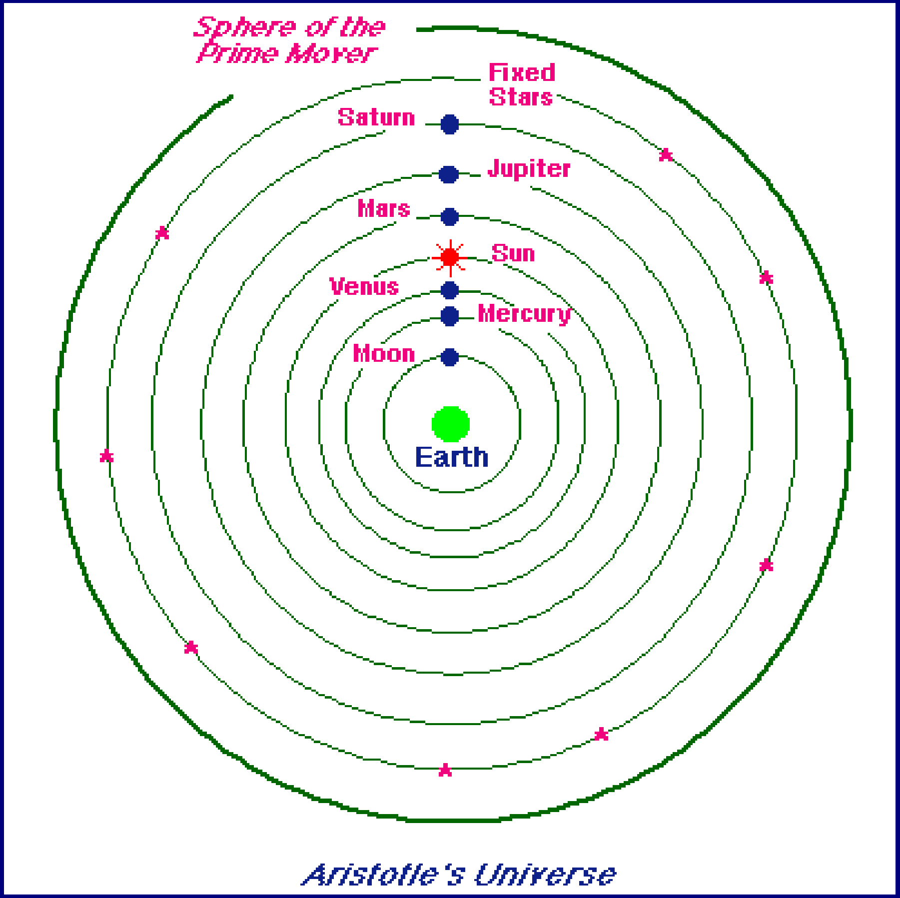
（4） $\vec{r} \cdot \vec{v} = \left[2t\vec{i} + \left(19 - 2t^2\right)\vec{j}\right] \cdot \left(2\vec{i} - 4t\vec{j}\right)$

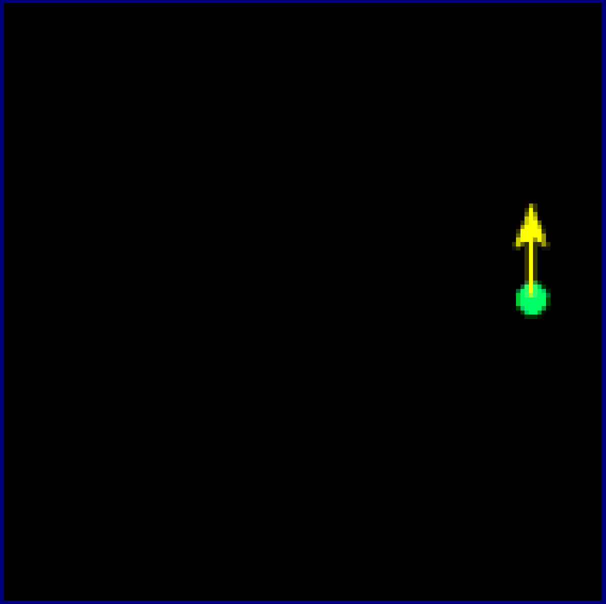
$$= 4t - 4t(19 - 2t^2) = 4t(2t^2 - 18)$$

$$= 8t(t+3)(t-3) = 0$$

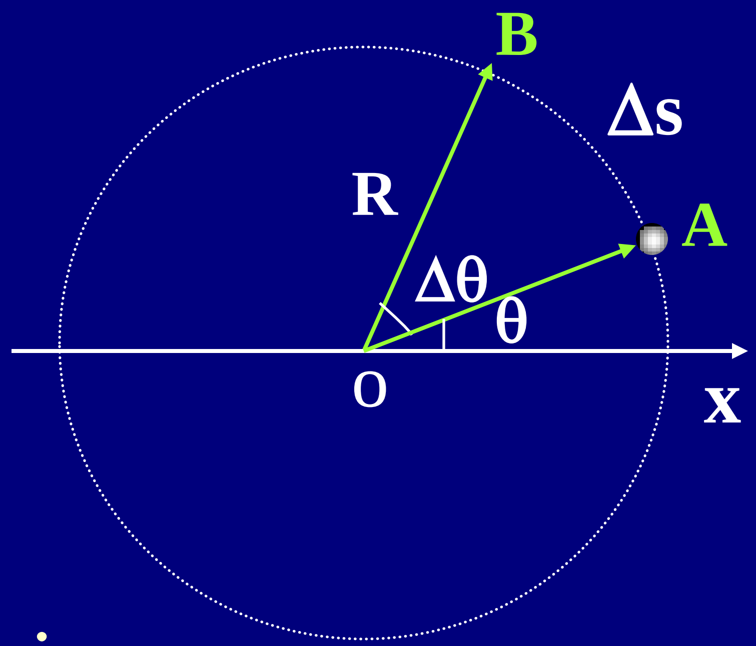
$$t_1 = 0\,(s) \quad , \quad t_2 = 3\,(s)$$

Circular movement





Uniform circular movement



Angular position θ :

Angular displacement:

When counterclockwise, Δθ is positive, when clockwise, Δθ is negative

Angular velocity ω:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \qquad (rad \cdot s^{-1})$$

Centripetal acceleration:

$$a = a_n = \frac{v^2}{r}$$

In general ways :

$$a_\tau = \frac{dv}{dt} \qquad a_n = \frac{v^2}{r}$$

Angular acceleration :

$$\beta = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \qquad (rad \cdot s^{-2})$$

Relationship between angular property and linear property

$$v = r\omega$$

$$a_{\tau} = r\beta$$

$$a_n = r\omega^2$$

Tangential acceleration:

$$\vec{a}_\tau = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}_\tau}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \vec{\tau} = \frac{dv}{dt} \vec{\tau}$$

Total acceleration:

$$\vec{a} = \vec{a}_n + \vec{a}_\tau$$

Magnitude:

$$\sqrt{a_n^2 + a_\tau^2}$$

Direction:

$$\theta = arctg \frac{a_n}{a_\tau}$$

The wheel in rotation has a radius of 0.2m， the angular displacement of a certain point on the edge of the wheel is given as φ=-t²+4t(rad)， what is the acceleration and velocity of P when t=1s。

solution:

$$\omega=\frac{d\varphi}{dt}=-2t+4 \qquad \beta=\frac{d\omega}{dt}=-2$$

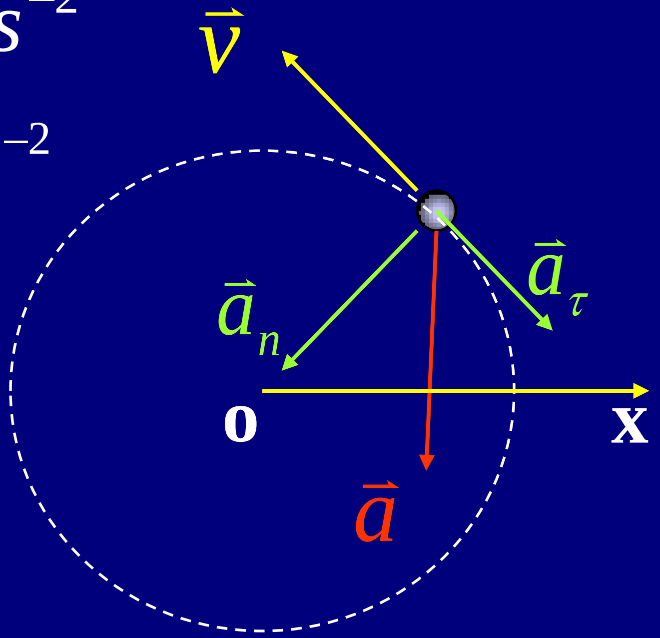
$$v=t\omega=r(-2t+4)=0.2\times(-2\times1+4)=0.4\,m\cdot s^{-1}$$

$$a_\tau=r\beta=0.2\times(-2)=-0.4\,m\cdot s^{-2}$$

$$a_n=r\omega^2=0.2(-2\times1+4)^2=0.8\,m\cdot s^{-2}$$

$$a=\sqrt{a_\tau^2+a_n^2}=0.89\,m\cdot s^{-2}$$

$$\theta=tg^{-1}\left|\frac{a_n}{a_\tau}\right|=tg^{-1}\frac{0.8}{0.4}=63.4^\circ$$



§ 1-5 Relative movement

$$\vec{v}_{bs} = \vec{v}_{bw} + \vec{v}_{ws}$$

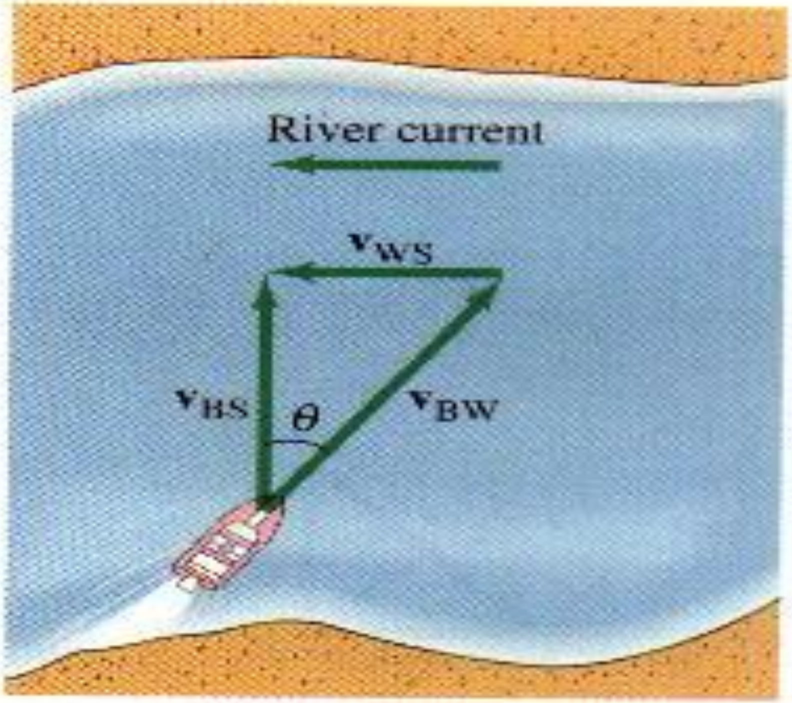


FIGURE 3–13 The boat must head upstream at an angle θ if it is to move directly across the river. Velocity vectors are shown as green arrows:

$\mathbf{v}_{BS}$ = velocity of **B**oat with respect to the **S**hore,

$\mathbf{v}_{BW}$ = velocity of **B**oat with respect to the **W**ater,

$\mathbf{v}_{WS}$ = velocity of the **W**ater with respect to the **S**hore.

A man is heading east with speed v on his bike, the wind is blowing from the direction of north 30 degree west with same speed. What direction will the man sense the wind is blowing from？

Solution：

$$\vec{v}_{wm} = \vec{v}_{wg} + \vec{v}_{gm} = \vec{v}_{wg} - \vec{v}_{mg}$$