Oscillations

Agenda Today

1 Simple harmonic motion

2 Pendulum

3 Damped harmonic motion

Oscillation(振动）：

The movement that repeat itself.

Simple harmonic motion:(简谐振动）

Any oscillation that could be expressed as a sinusoidal function

$$x = A\cos(\omega t + \varphi)$$

A： amplitude(振幅）

φ：phase constant(振动的“初位相”)

（ω t+φ）：phase(振动的“位相”)

ω：angular frequency(圆频率）

Frequency （频率）ν：the number of oscillations that finished in one second.

$$\nu = \frac{\omega}{2\pi}$$

Period （周期）T：the time needed to finish one oscillation

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}$$

Velocity and acceleration of SHM

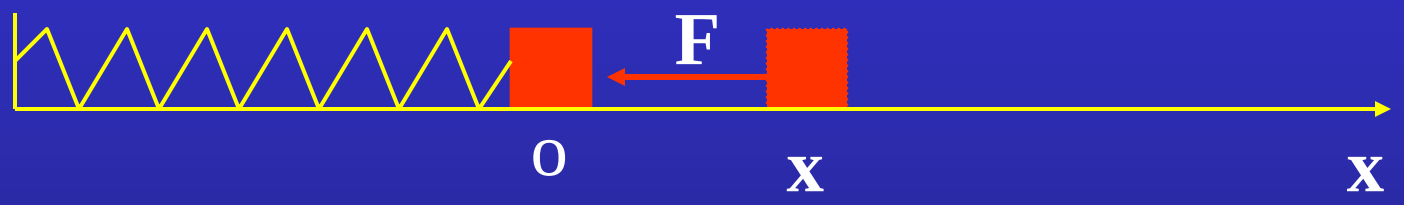
$$v=\frac{dx}{dt}=-\omega A\sin(\omega t+\varphi)=v_m\cos(\omega t+\varphi+\frac{\pi}{2})$$

$$a=\frac{dv}{dt}=-\omega^2 A\cos(\omega t+\varphi)=a_m\cos(\omega t+\varphi\pm\pi)$$

1、$v_m=\omega A$ velocity amplitude“速度幅 ”。

2、$a_m=\omega^2 A$ acceleration amplitude “加速度幅 ”。

Spring oscillator（弹簧振子）



By Hook's law

$$\vec{F} = -k\vec{x}$$

By Newton's second law of motion:

$$F = m\frac{d^2x}{dt^2} = -k\,x$$

that is

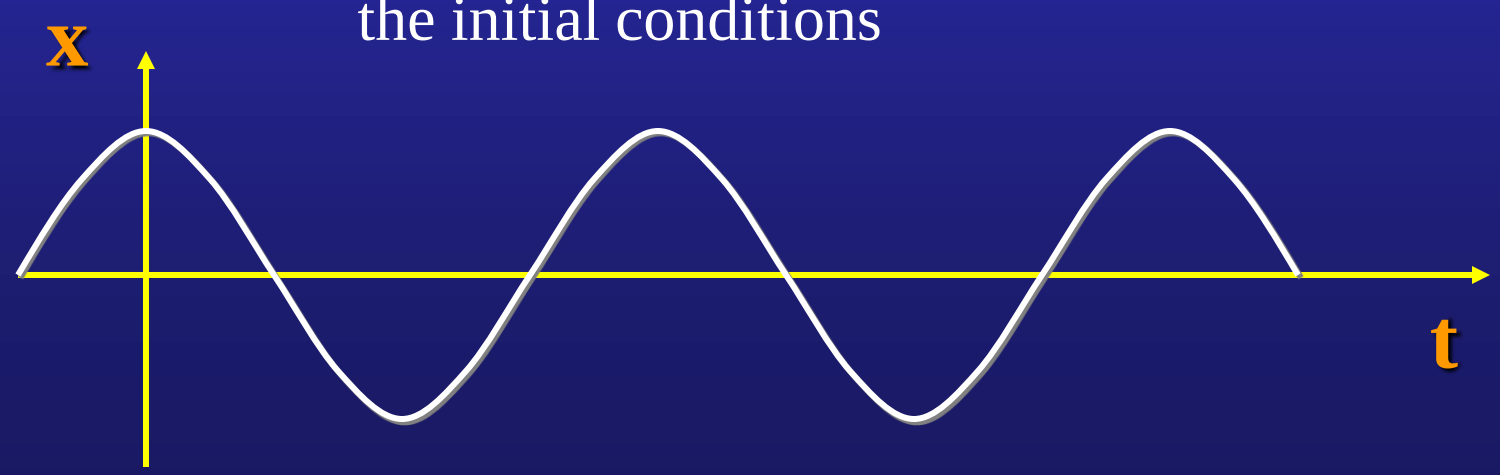
$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

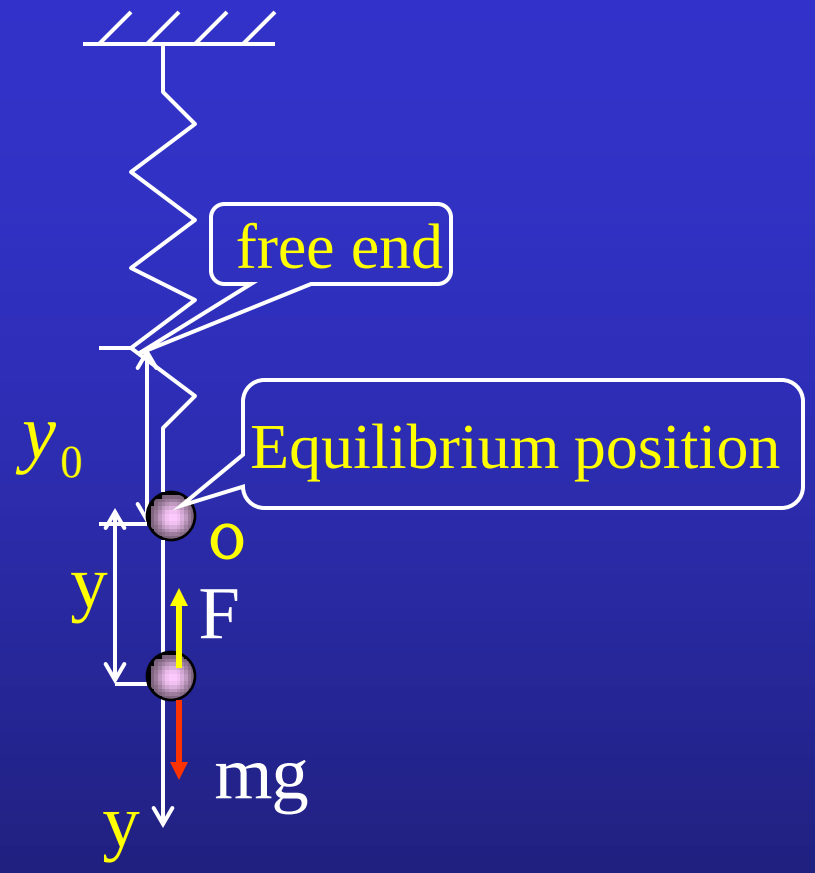
$$\omega = \sqrt{\frac{k}{m}}$$

The solution:

$$x = A\cos(\omega t + \varphi)$$

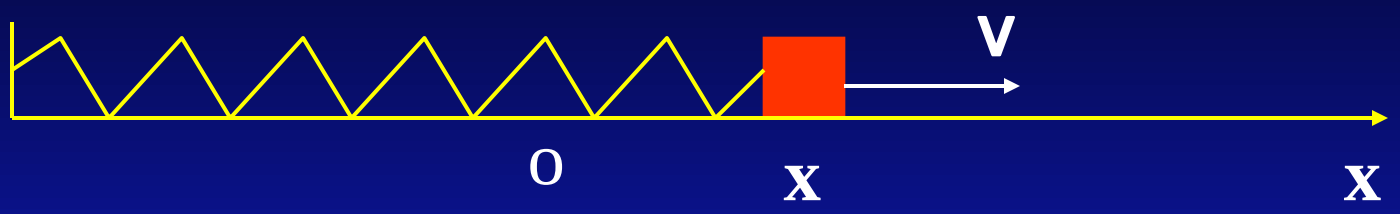
Where A, and ϕ are decided by the initial conditions





Does the oscillator has same period when it hangs from the ceiling?

The energy of SHM



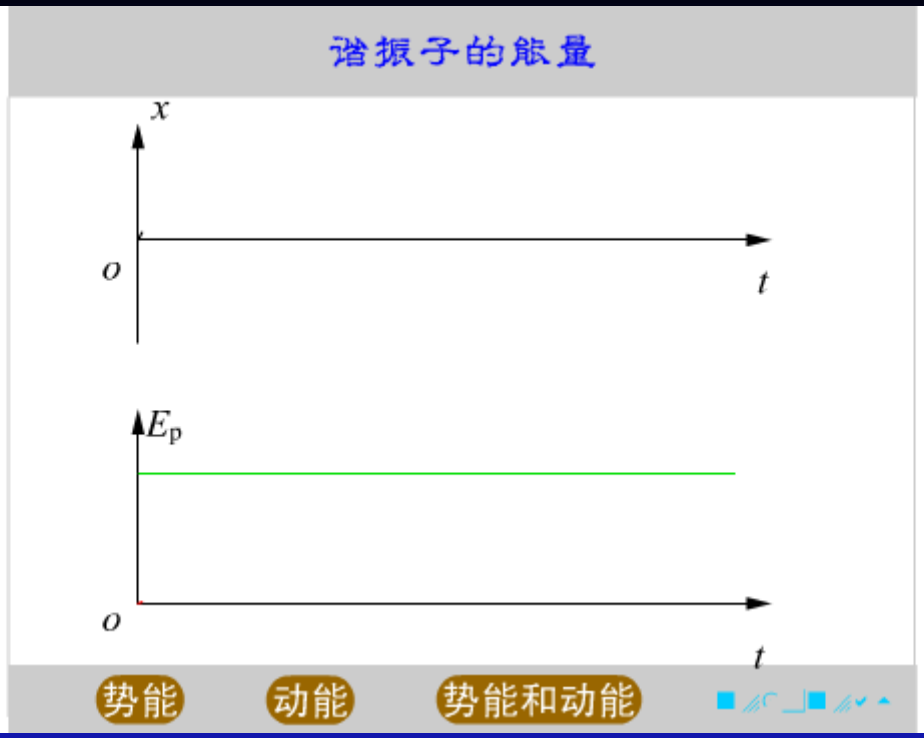
Kinetic energy:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \varphi)$$

Potential energy:

$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \varphi)$$

$$m\omega^2 = k$$



$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \varphi)$$

$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \varphi)$$

$$E = E_k + E_p$$

$$= \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \varphi) + \frac{1}{2} kA^2 \cos^2(\omega t + \varphi)$$

$$E = \frac{1}{2} kA^2 = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m v_m{}^2$$

The total energy of SHM does not change with time

Simple pendulum(单摆)

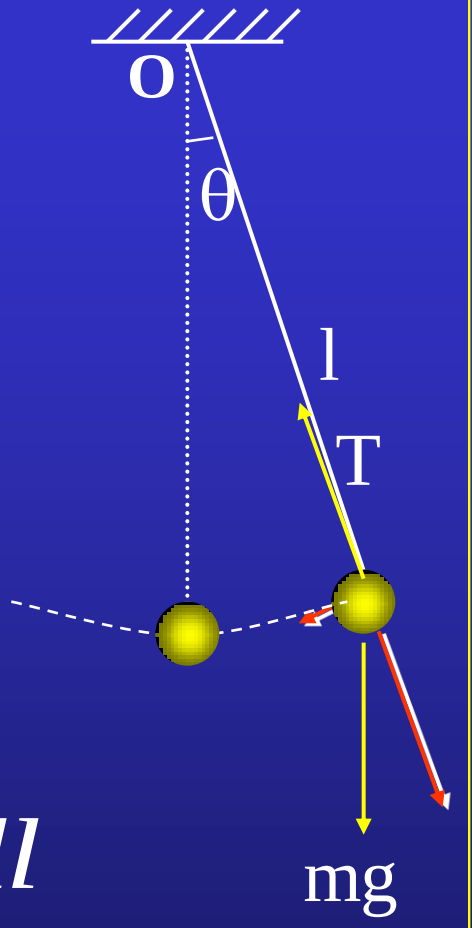
$$-mg\sin\theta = m\frac{d^2s}{dt^2}$$

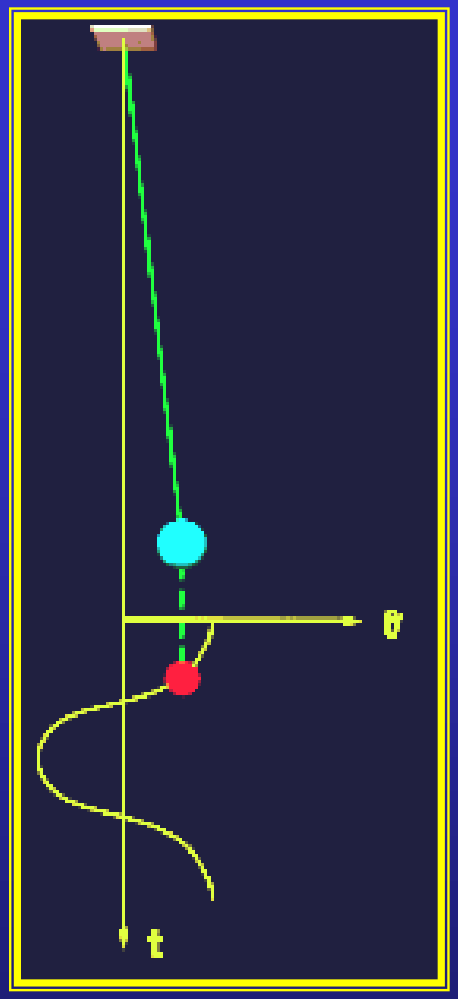
$$\because \quad s = l\theta$$

$$-mg\sin\theta = ml\frac{d^2\theta}{dt^2}$$

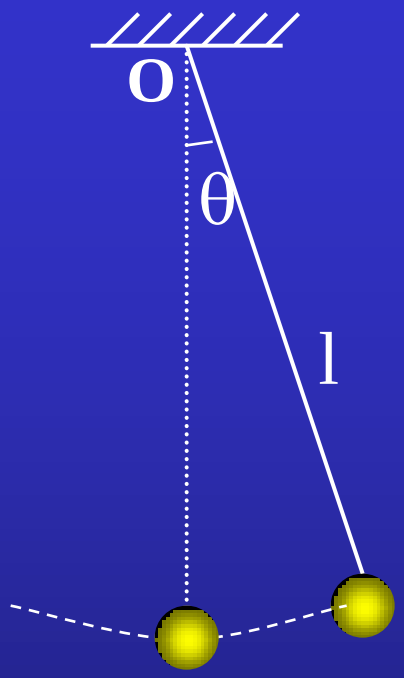
$$if \quad \theta \ \ is\ very\ small$$

$$\therefore \sin\theta \approx \theta$$





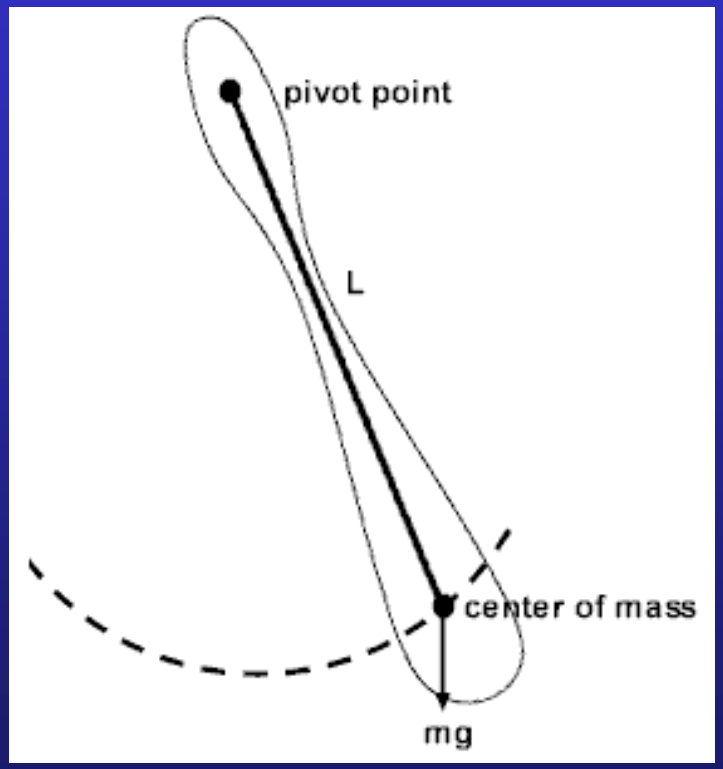
$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$



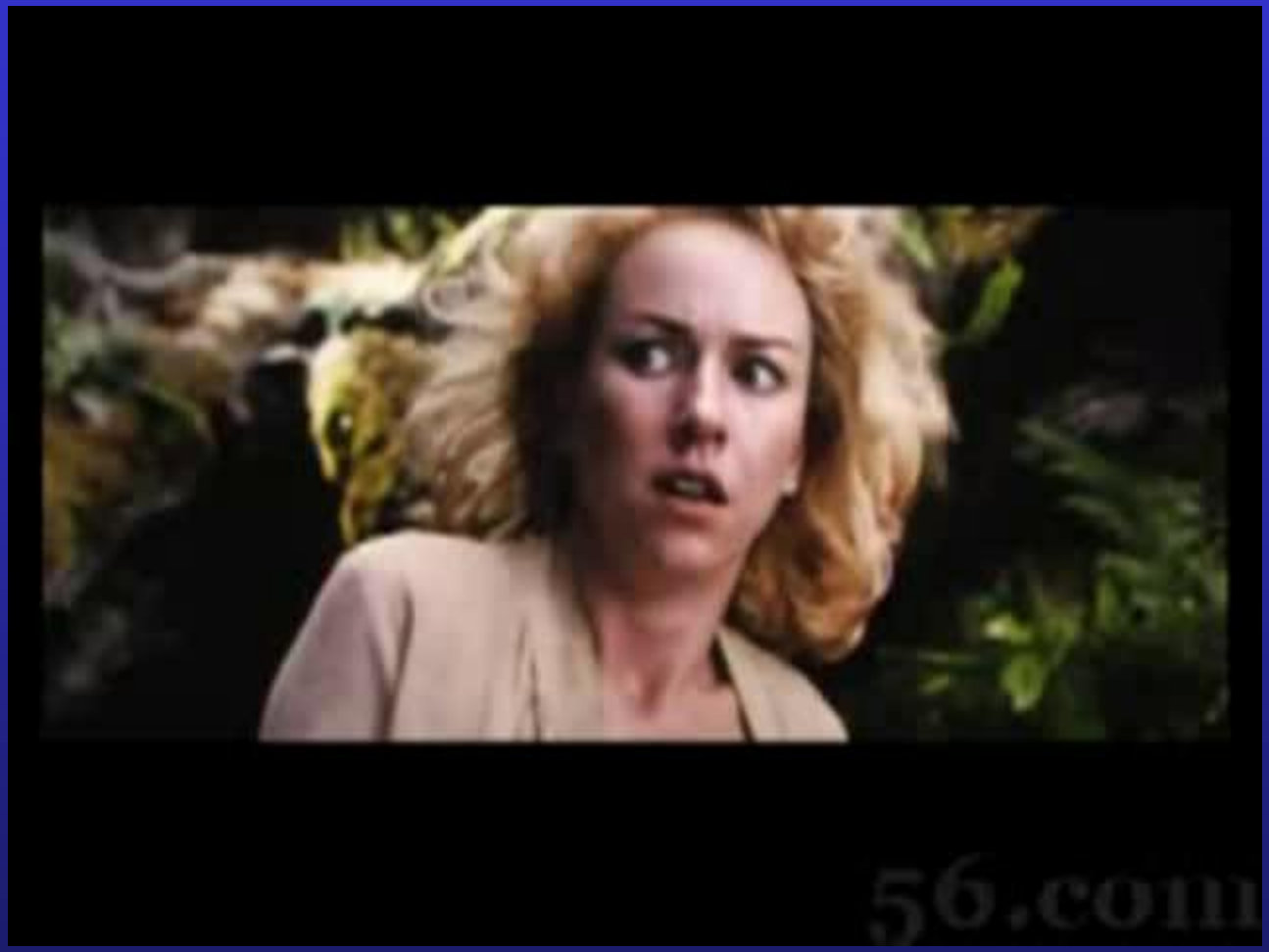
$$\omega = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Physical pendulum(复摆）



$$T = 2\pi\sqrt{\frac{I}{mgL}}$$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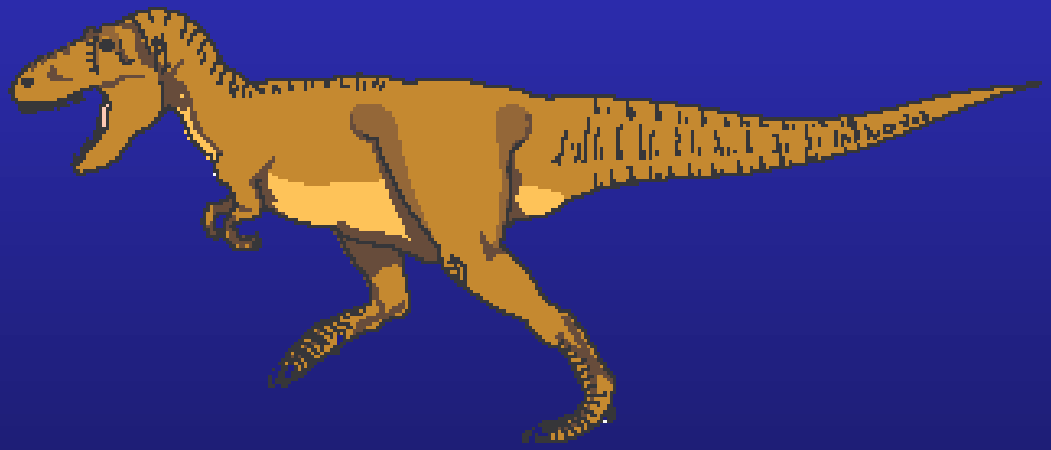
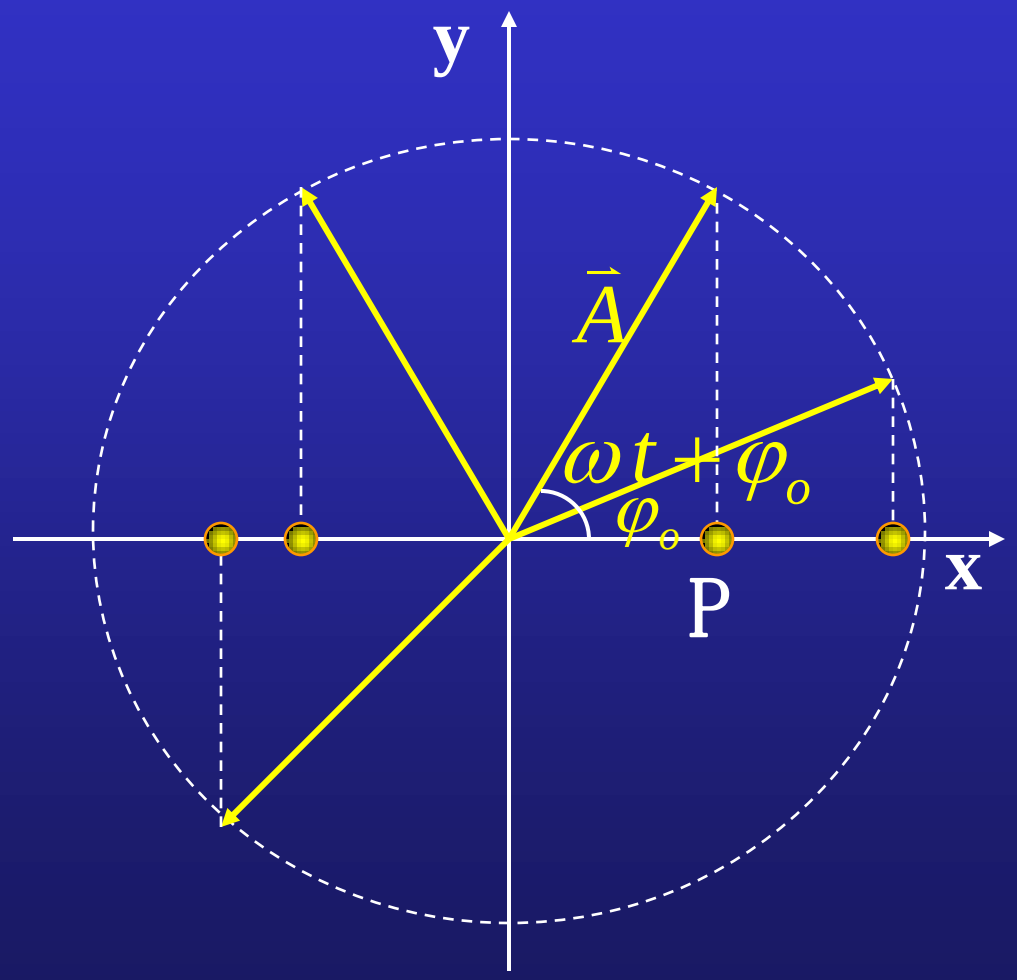
How Fast Was Tyrannosaurus Rex Traveling?



Similarity between SHM and circular motion

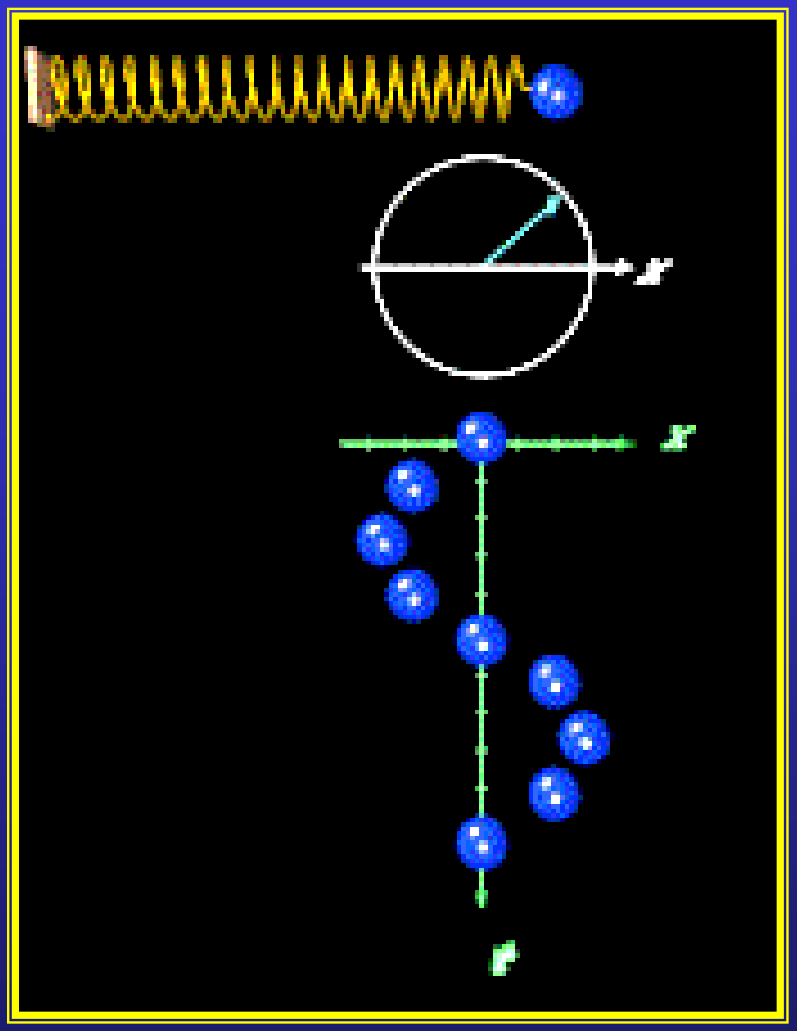
$$x = A\cos(\omega t + \varphi)$$

the projection on a straight line of a particle moving with uniform circular motion is simple harmonic motion



简谐振动的矢量图示

play pause



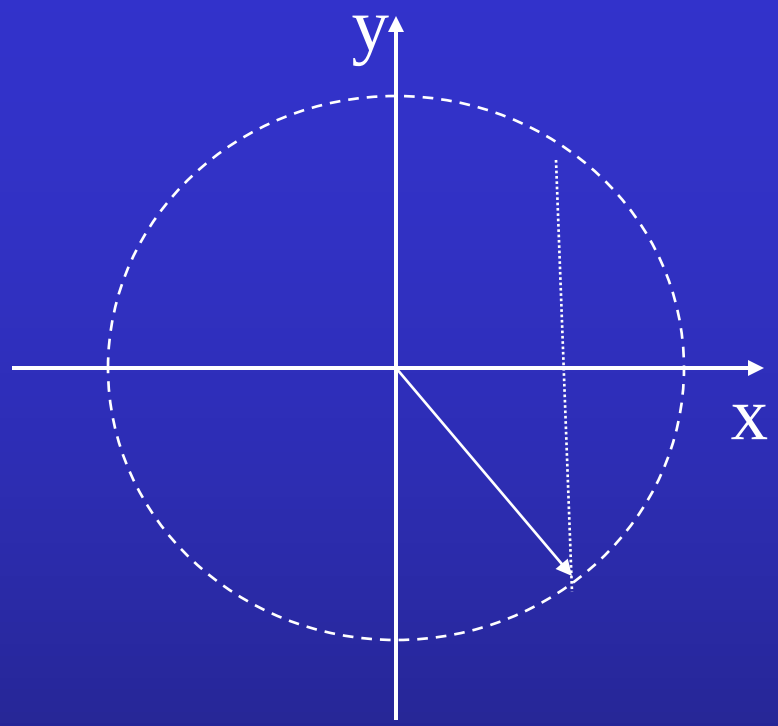
Example: A particle in SHM moves along x-axis, its amplitude is 12cm, the period of SHM is 2s。 when t=0, its displacement is 6 cm and it moves towards its positive maximum, find: a) the equation that describes the motion Of the SHM, B) when will it arrive its equilibrium position.

Solution: **Assume the equation is:** $X = A\cos(\omega t+\varphi)$

A=12cm , T=2s ,

$$\omega = \frac{2\pi}{T} = \pi\, s^{-1}$$

$$X=0.12\cos(\pi t + \varphi)$$

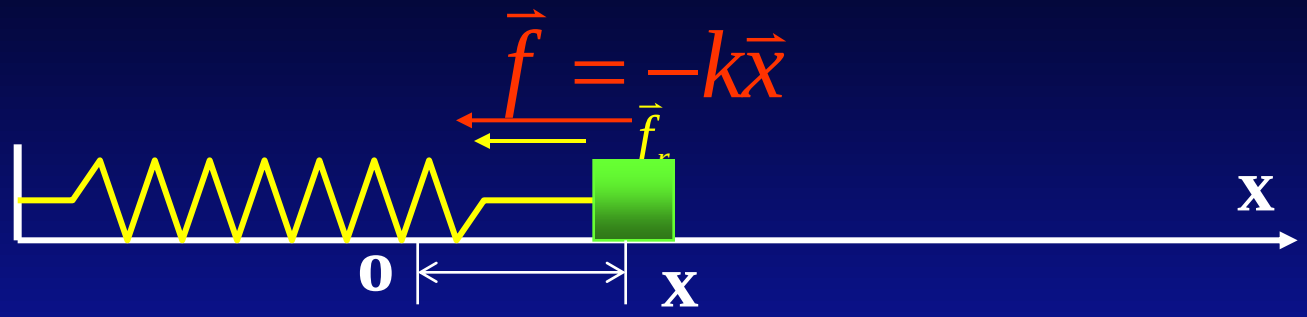


Oscillation equation:

$$x = 0.12\cos(\pi t - \frac{\pi}{3})$$

$$\Delta t = \frac{\Delta\varphi}{\omega} = \frac{5}{6}\pi / \pi = \frac{5}{6}s$$

Damped oscillation (阻尼振动)



$$f_r = -\mu v$$

$$-kx - \mu v = m\frac{d^2x}{dt^2}$$

$$Let: \quad \omega_o^2 = \frac{k}{m} \quad , \quad 2\beta = \frac{\mu}{m}$$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_o{}^2 x = 0$$

1、 slightly damped oscillation $\beta^2 < \omega_0{}^2$

2. Critically damped oscillation ($\beta=\omega_0$)

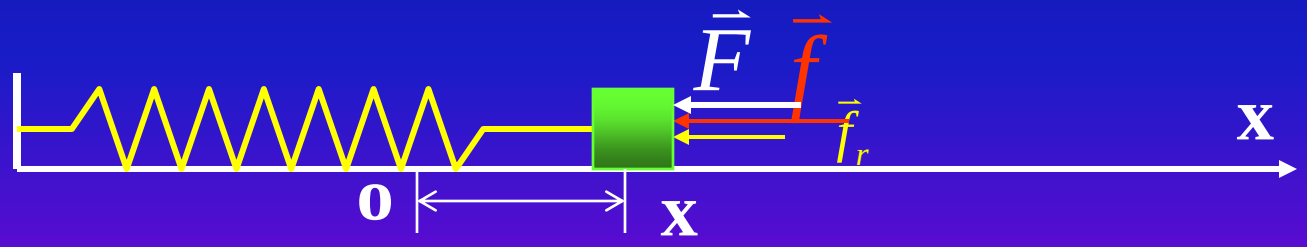
3. Over-damped oscillation ($\beta>\omega_0$)

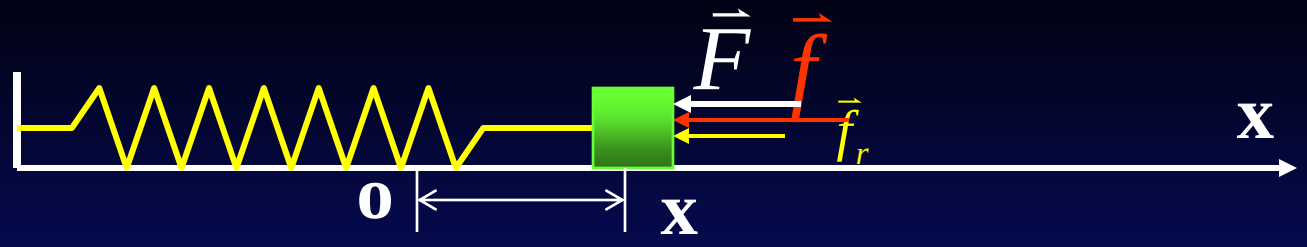
Driven oscillation(受迫振动）

Driving force：

The periodical force exert on the oscillator

let： $$F = F_o \sin \omega t$$





$$m\frac{d^2x}{dt^2} = -kx - \mu\frac{dx}{dt} + F_o \sin\omega t$$

let: $\frac{k}{m} = \omega_o^2 \qquad \frac{\mu}{m} = 2\beta \qquad \frac{F_o}{m} = f$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_o^2 x = f\sin\omega t$$

$$x = Ae^{-\beta t}\cos(\sqrt{\omega_o^2 - \beta^2}t + \varphi) + B\sin(\omega t - \Delta\varphi)$$

Steady state solution：

$$x = B\sin(\omega t - \Delta\varphi)$$

$$B = \frac{f}{\sqrt{\left(\omega_o^2 - \omega^2\right)^2 + 4\beta^2\omega^2}}$$

$$tg\Delta\varphi = \frac{2\beta\omega}{\left(\omega_o^2 - \omega^2\right)}$$

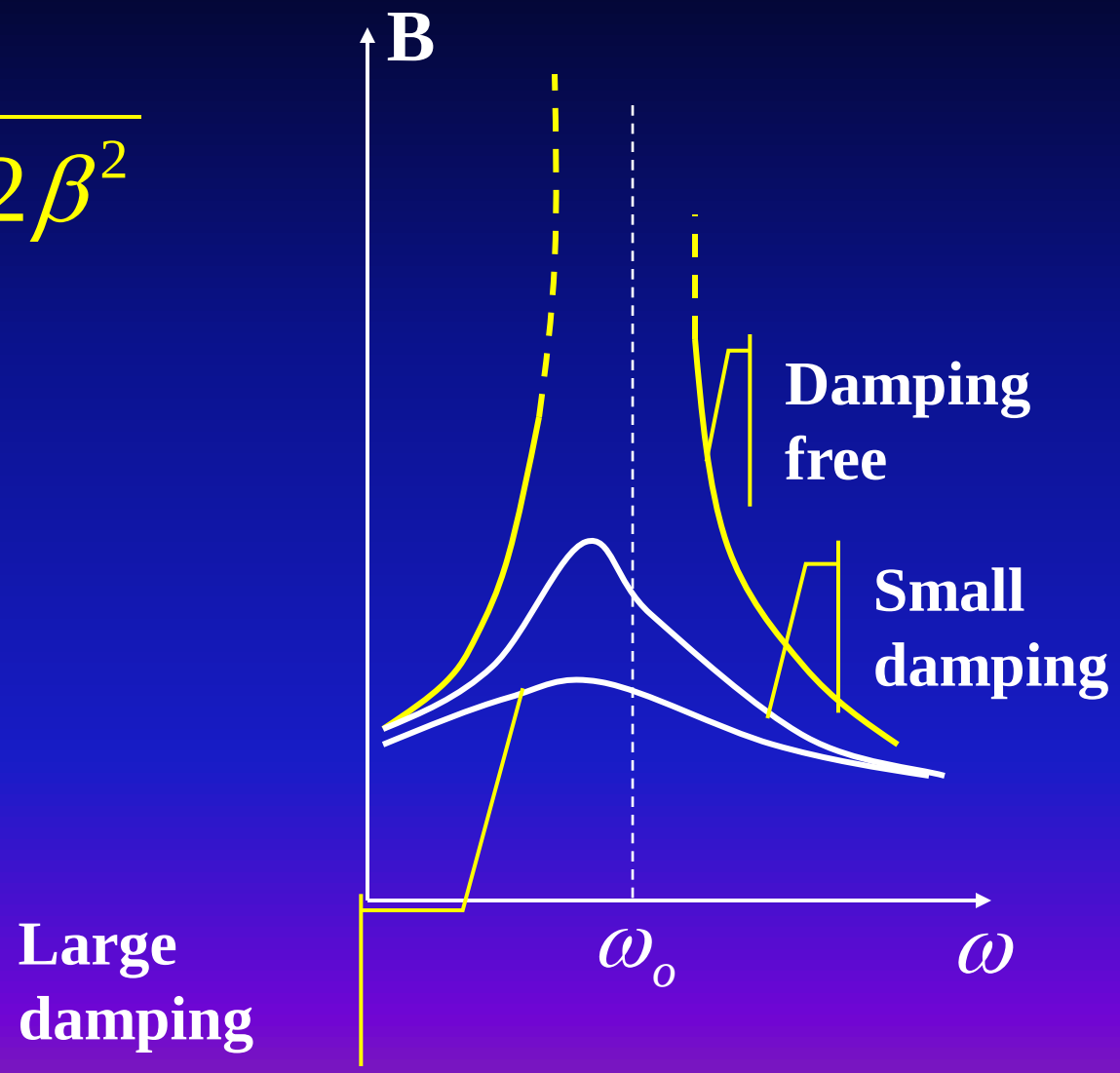
the frequency of driven oscillator in steady state is that of the driving force。

Resonance（共振）

when the frequency of the driving force reaches a certain value, the amplitude of the oscillator reaches its peak value.

$$B = \frac{f}{\sqrt{\left(\omega_o^2 - \omega^2\right)^2 + 4\beta^2\omega^2}}$$

$$\frac{dB}{d\omega} = \frac{d}{d\omega}\left(\frac{f}{\sqrt{\left(\omega_o^2 - \omega^2\right)^2 + 4\beta^2\omega^2}}\right) = 0$$

The resonance frequency:

$$\omega_r = \sqrt{\omega_o^2 - 2\beta^2}$$