Mechanical Waves

Agenda today

1 Transverse waves and longitudinal waves

2 The speed of waves,

3 wavelength, frequency, phase

4 wave function

Particle and waves

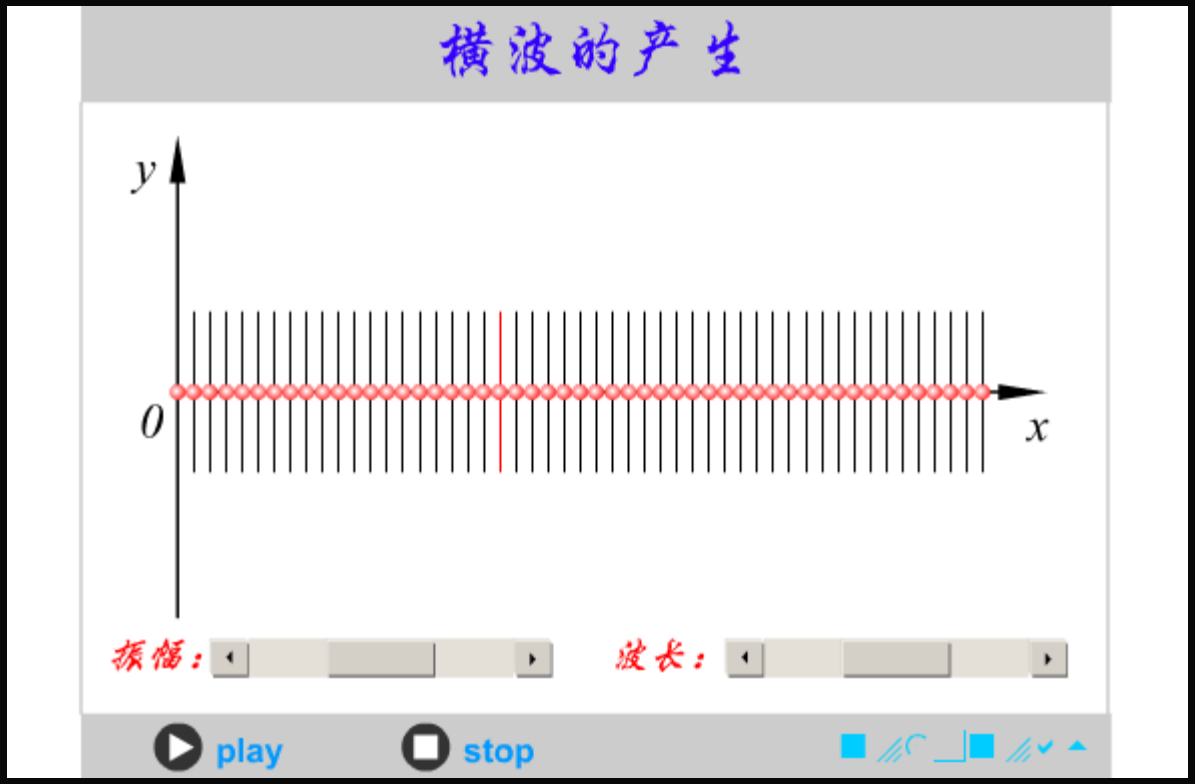
Type of waves:

Mechanical Waves（机械波）

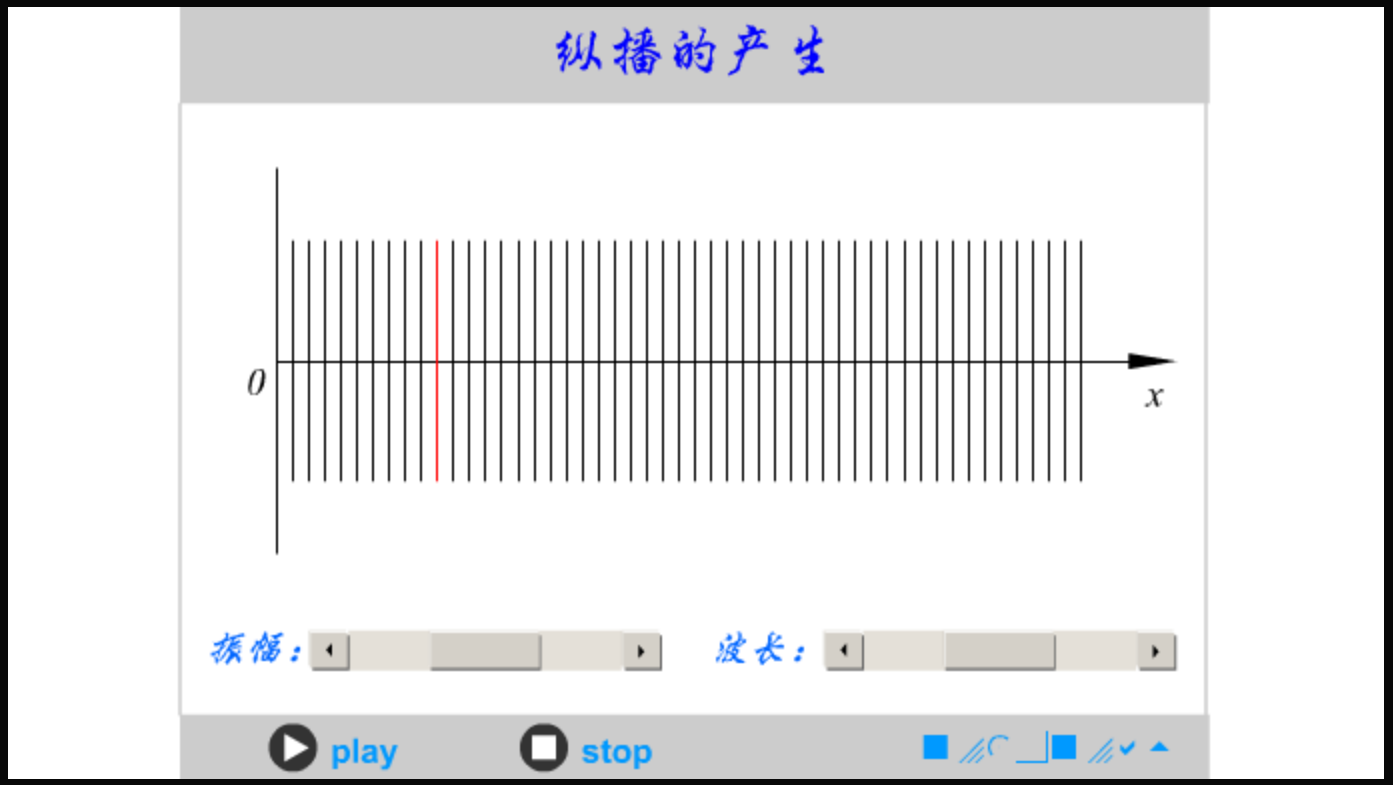
Electromagnetic waves（电磁波）

Matter waves（物质波）

Transverse waves and longitudinal waves（横波和纵波）



Transverse waves : the direction of the particle’s motion is perpendicular to the direction that waves propagate.



Longitudinal waves: the direction of particle's motion is the same direction that the waves travel.

Surface waves（表面波）：

Water wave breaks when it approach the beach

The function for sinusoidal wave:

$$y(x,t) = A\cos[\frac{2\pi}{\lambda}(x \pm ut) + \varphi]$$

Where + means waves travel left;

- means waves travel right with speed u.

Some concepts about waves:

Amplitude(振幅）： the maximum displacement that a particle can make in the medium.

Peak and Valley(波峰与波谷）：

Wavelength（波长）:the minimum distance between any identical point along the direction of wave propagation

Period（周期）： the period of the particle in oscillation

Frequency（频率）:the number of peaks (valleys) that pass a given point in a unit time interval.

$$\boxed{\nu = \frac{1}{T}}$$

Angular frequency:

$$\omega = \frac{2\pi}{T}$$

Angular wave number:

$$k = \frac{2\pi}{\lambda}$$

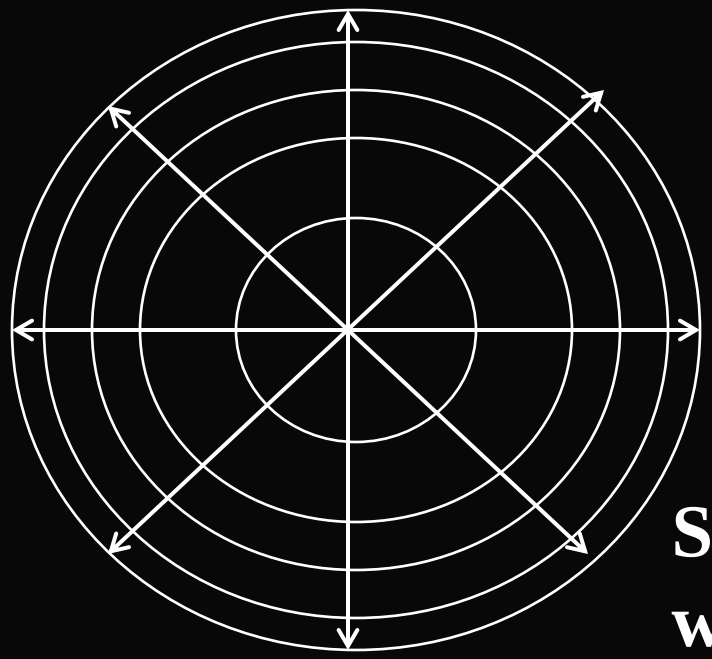
Speed of waves: the distance that waves travel in unit time.

$$\boxed{u = \frac{\lambda}{T} = \nu\lambda = \frac{\omega}{k}}$$

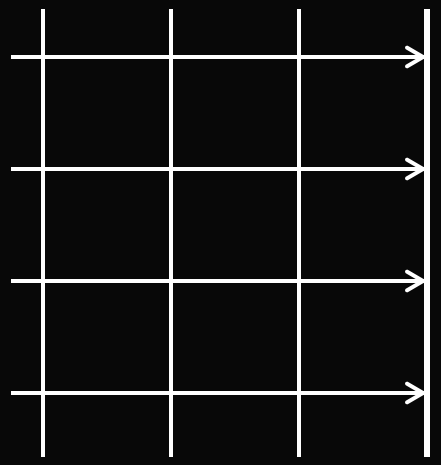
Phase:

Wave surface（波阵面）： ***the surface on which all the particles have the same phase***

Wave front（波前）: ***the first wave surface that the wave propagates***



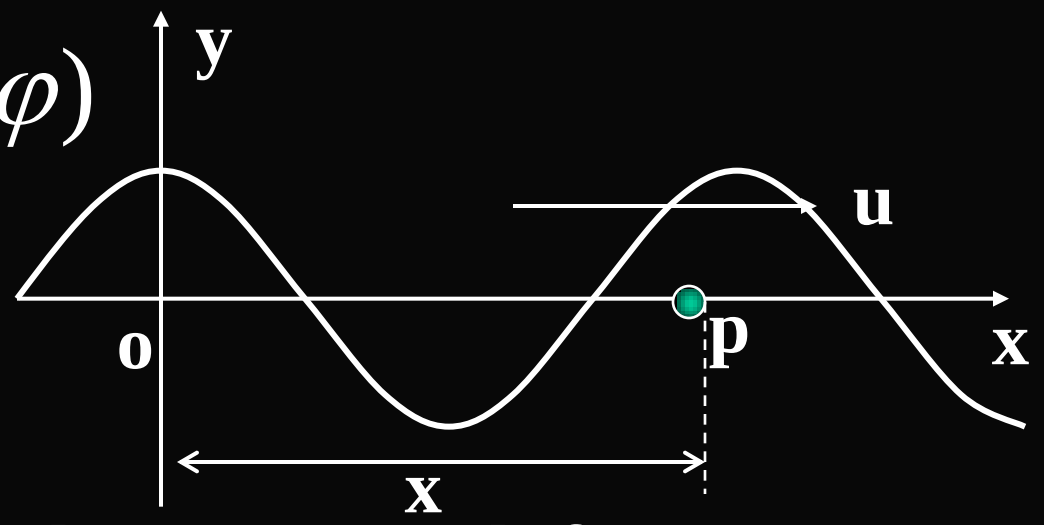
Spherical waves



Plane waves

The equation that describes the motion of particle o:

$$y_0 = A\cos(\omega t + \varphi)$$



The time needed that the wave travels from o to p is:

$$\Delta t = \frac{x}{u}$$

$$y_p = A\cos[\omega\,(t - \frac{x}{u}) + \varphi] = A\cos[(\omega t - \omega\frac{x}{u}) + \varphi]$$

$$y(x,t) = A\cos[\omega\ (t - \frac{x}{u}) + \varphi]$$

$$A\cos\omega\ (t - \frac{x}{u}) = A\cos\frac{2\pi}{T}(t - \frac{x}{u})$$

$$= A\cos 2\pi(\frac{t}{T} - \frac{x}{\lambda})$$

$$y(x,t) = A\cos[2\pi(\frac{t}{T} - \frac{x}{\lambda}) + \varphi]$$

$$A\cos 2\pi(\frac{t}{T}-\frac{x}{\lambda}) = A\cos 2\pi(\frac{x}{\lambda}-\frac{t}{T})$$

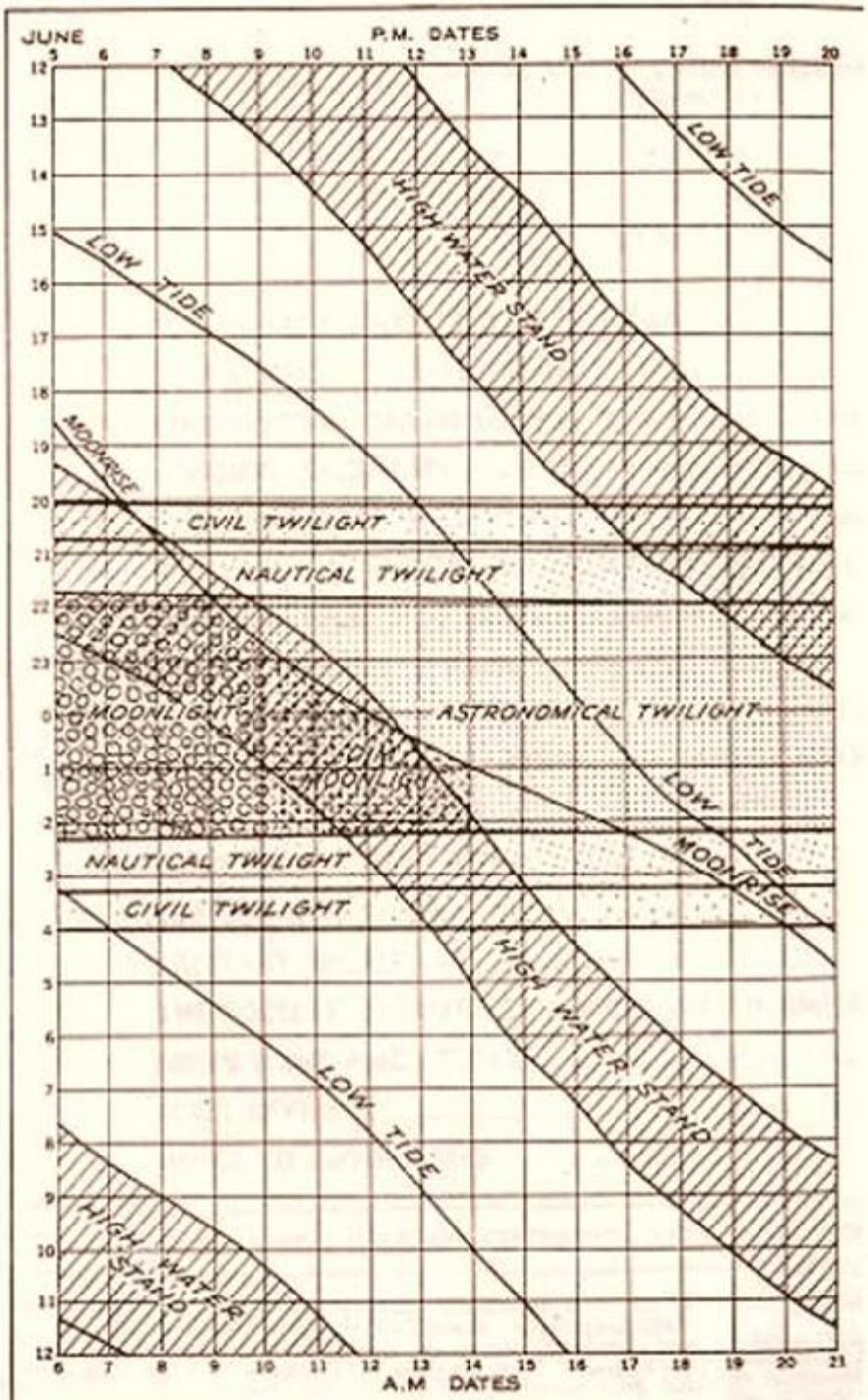
$$= A\cos\frac{2\pi}{\lambda}(x-ut) = A\cos k(x-ut)$$

$$= A\cos(kx-\omega t)$$

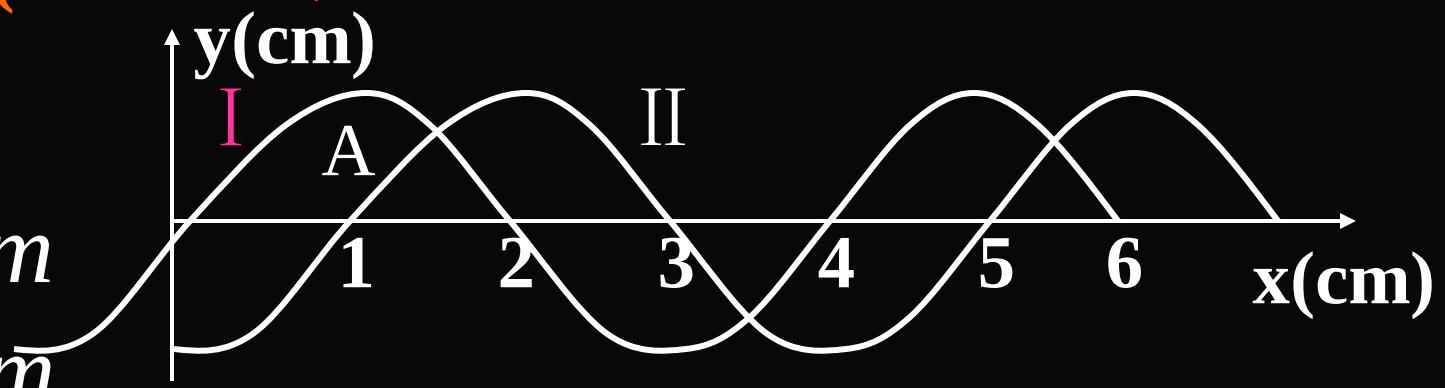
$$\boxed{y(x,t) = A\cos(kx-\omega t+\varphi)}$$

K: wave number

at many locations energy is found at three semidiurnal frequencies: 1.93 cycles per day (due to the Moon), 2.00 cycles per day (due to the Sun), and 1.90 cycles per day (due to the ccentricity of the lunar orbit).


JUNE
P.M. DATES
A.M DATES
LOW TIDE
HIGH WATER STAND
MOONRISE
CIVIL TWILIGHT
NAUTICAL TWILIGHT
ASTRONOMICAL TWILIGHT
MOONLIGHT
DAY
NIGHT

The wave form at t=0 is Ⅰ, and the waves travel rightwards, after 1/2s its form changes to Ⅱ. The amplitude is 0.01m,The period T>1s, find the equation that describes the wave and particle A's movement (A=0.01m)



solution:

$$A = 0.01m$$

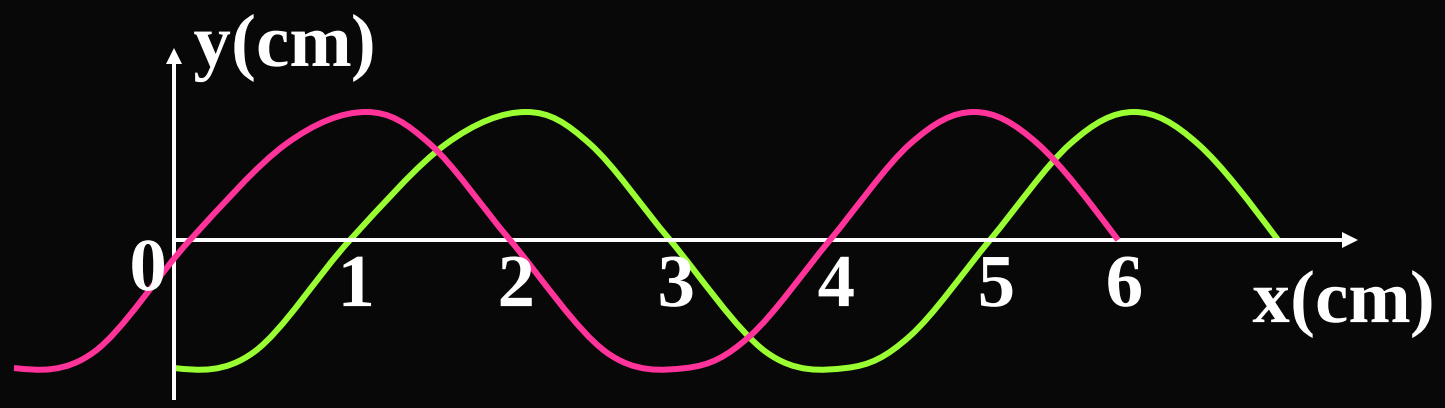
$$\lambda = 0.04m$$

The speed of wave:

$$u = \frac{x_1 - x_o}{t} = \frac{0.01}{1/2} = 0.02m \cdot s^{-1}$$

$$T = \frac{\lambda}{u} = \frac{0.04}{0.02} = 2s$$

$$\omega = \frac{2\pi}{T} = \pi\, s^{-1}$$



Assume the oscillation of o: $y_o = A\cos(\omega t + \varphi)$

Initial condition: $0 = A\cos\varphi \quad \rightarrow \varphi = \pm\dfrac{\pi}{2}$

$u = -\omega A\sin\varphi < 0 \qquad \sin\varphi > 0 \quad \rightarrow \varphi = \dfrac{\pi}{2}$

$$y_o = 0.01\cos(\pi t + \frac{\pi}{2})$$

$$y_o = 0.01\cos(\pi t + \frac{\pi}{2})$$

The equation describing the wave :

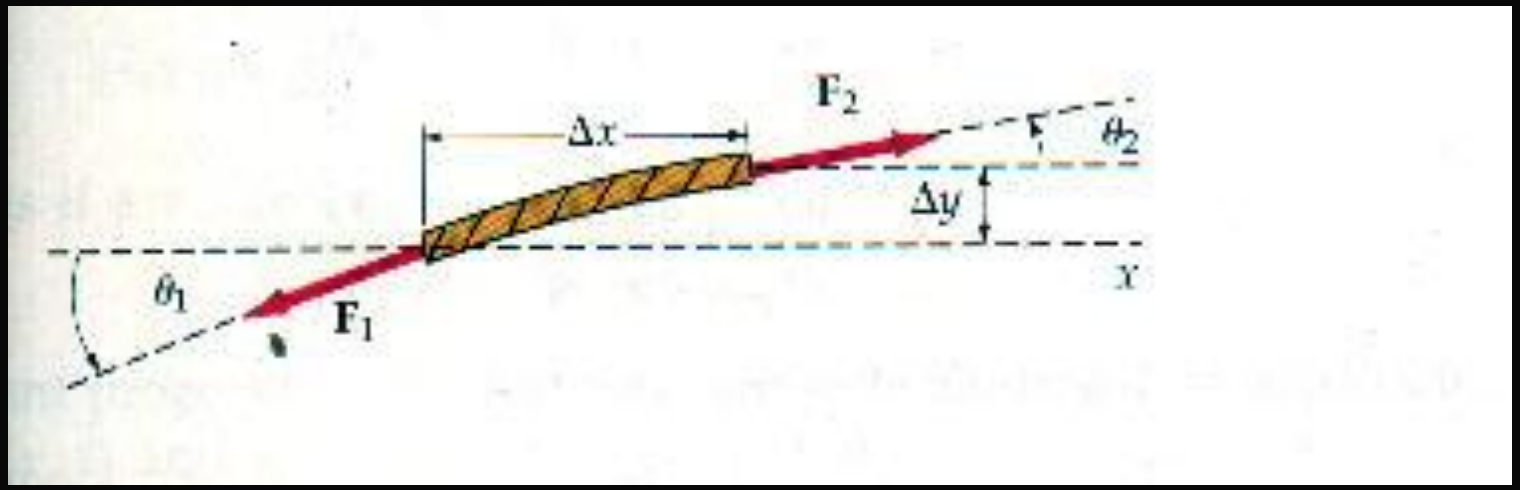
$$y = 0.01\cos[(\pi\ t - 50\pi x) + \frac{\pi}{2}]$$

The equation describing the motion of particle A:

$$y_A = 0.01\cos[\pi(t - \frac{0.01}{0.02}) + \frac{\pi}{2}]$$

$$y_A = 0.01\cos\pi\ t$$

The equation of waves for harmonic waves:



choose one segment of rope, if the amplitude is small

$$\sum F = F\sin\theta_2 - F\sin\theta_1 \approx F(\tan\theta_2 - \tan\theta_1)$$

$$= F\Delta x\frac{\partial^2 y}{\partial x^2}$$

$$ma = \mu \Delta x \frac{\partial^2 y}{\partial t^2}$$

By Newton's second law of motion:

F=ma

We find the PDE that describe the motion the rope waves

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2} \frac{\partial^2 y}{\partial t^2}$$

$$u = \sqrt{F / \mu}$$

speed of waves in rope

$$u = \sqrt{\frac{F}{\mu}}$$

F :tension， μ: mass density of the rope。

Speed of longitudinal waves:

In liquid and gas

$$u = \sqrt{\frac{B}{\rho}}$$

B: volume modules

In solid:

$$u = \sqrt{\frac{Y}{\rho}}$$

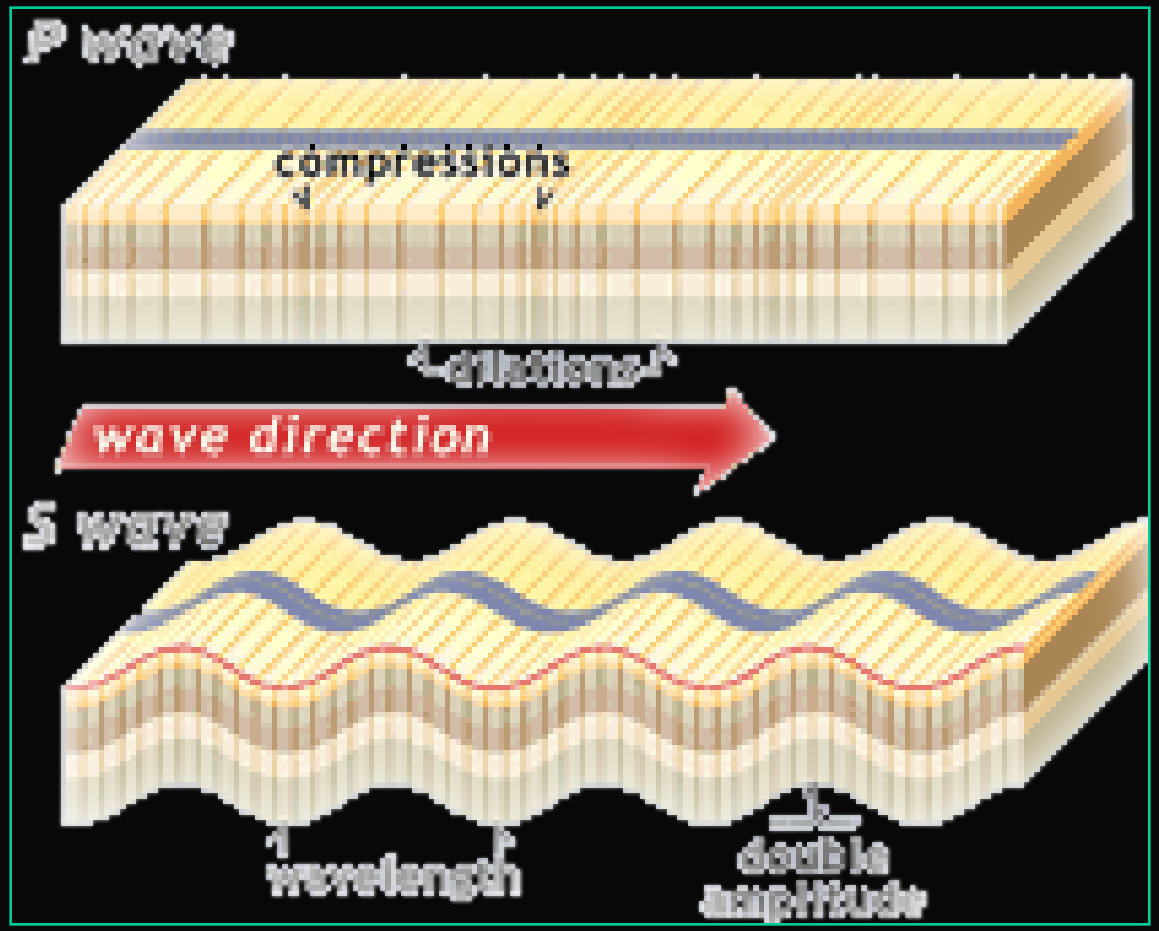
Y: Young's modules

Speed of transverse waves

$$u = \sqrt{\frac{G}{\rho}}$$

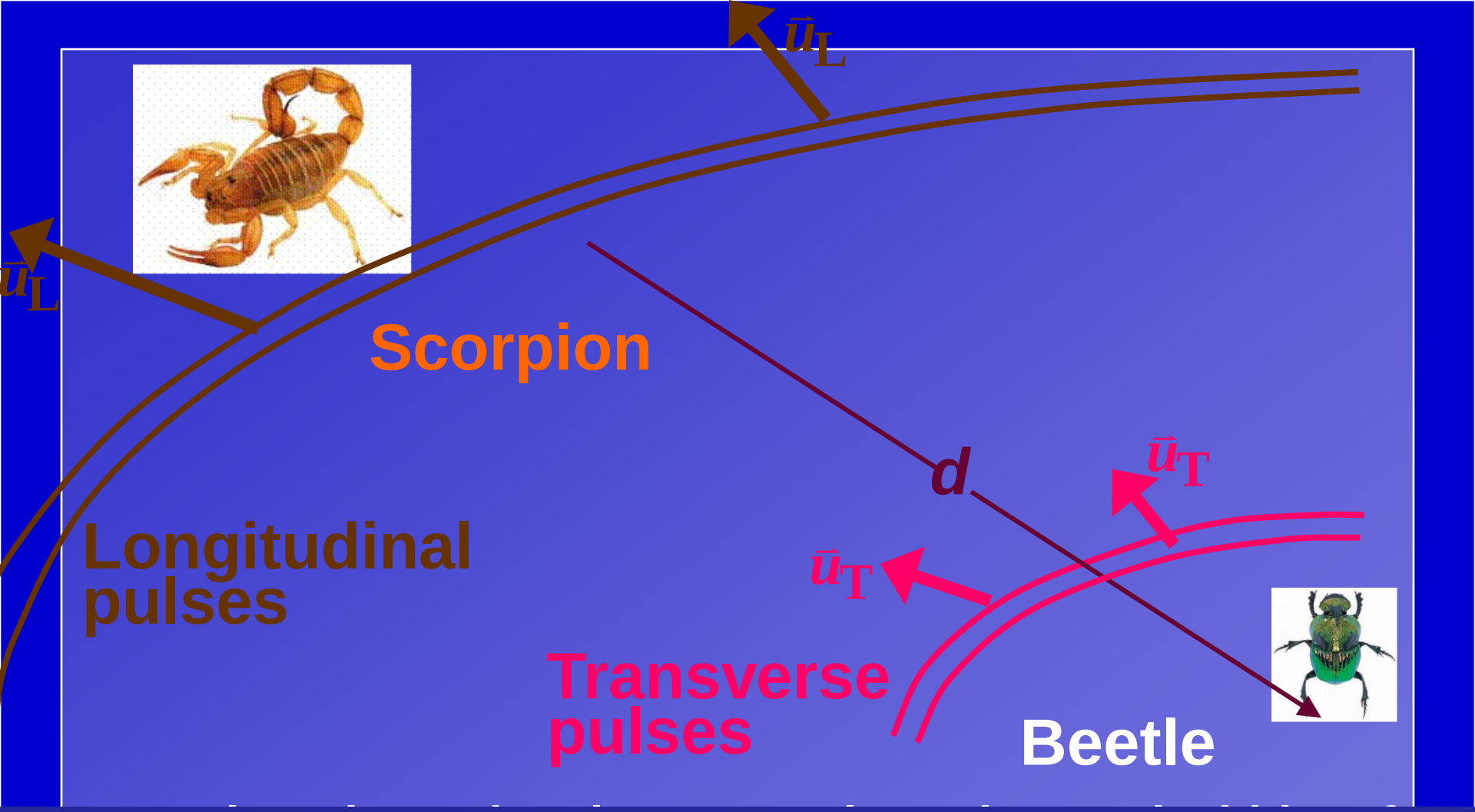
G: shear modules

Transverse waves can only travel in solid.



How to find the source of earthquake

Subadult Smeringerus vachoni
vs.
3" CA centipede



Example: When a beetle moves along the sand within a few tens of centimeters of this sand scorpion, the scorpion immediately turns toward the beetle and dash to it. The scorpion can do this without seeing the beetle, how?