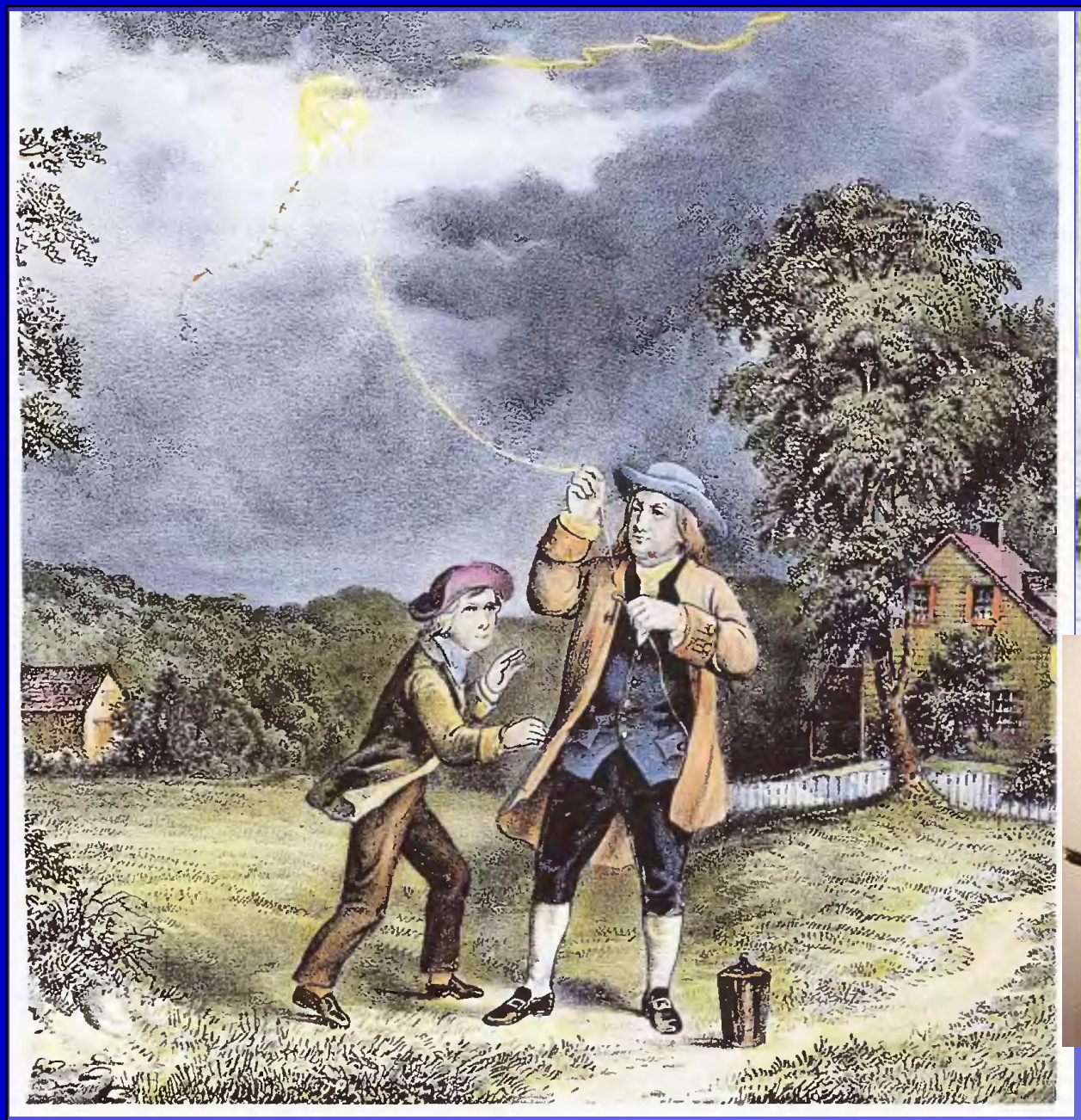
Electromagnetism

Agenda today:

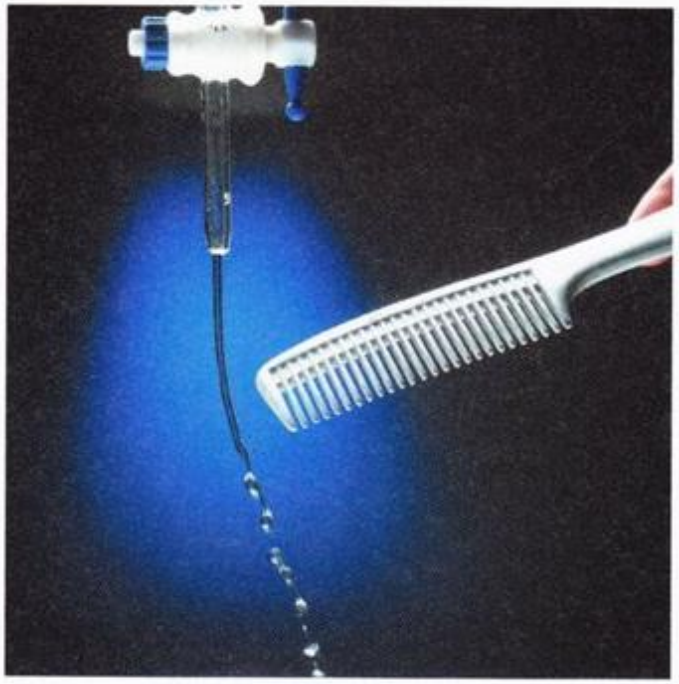
Electric charge（电量）

Coulomb’s Law（库仑定律）

Electric field（电场）



Charging by friction

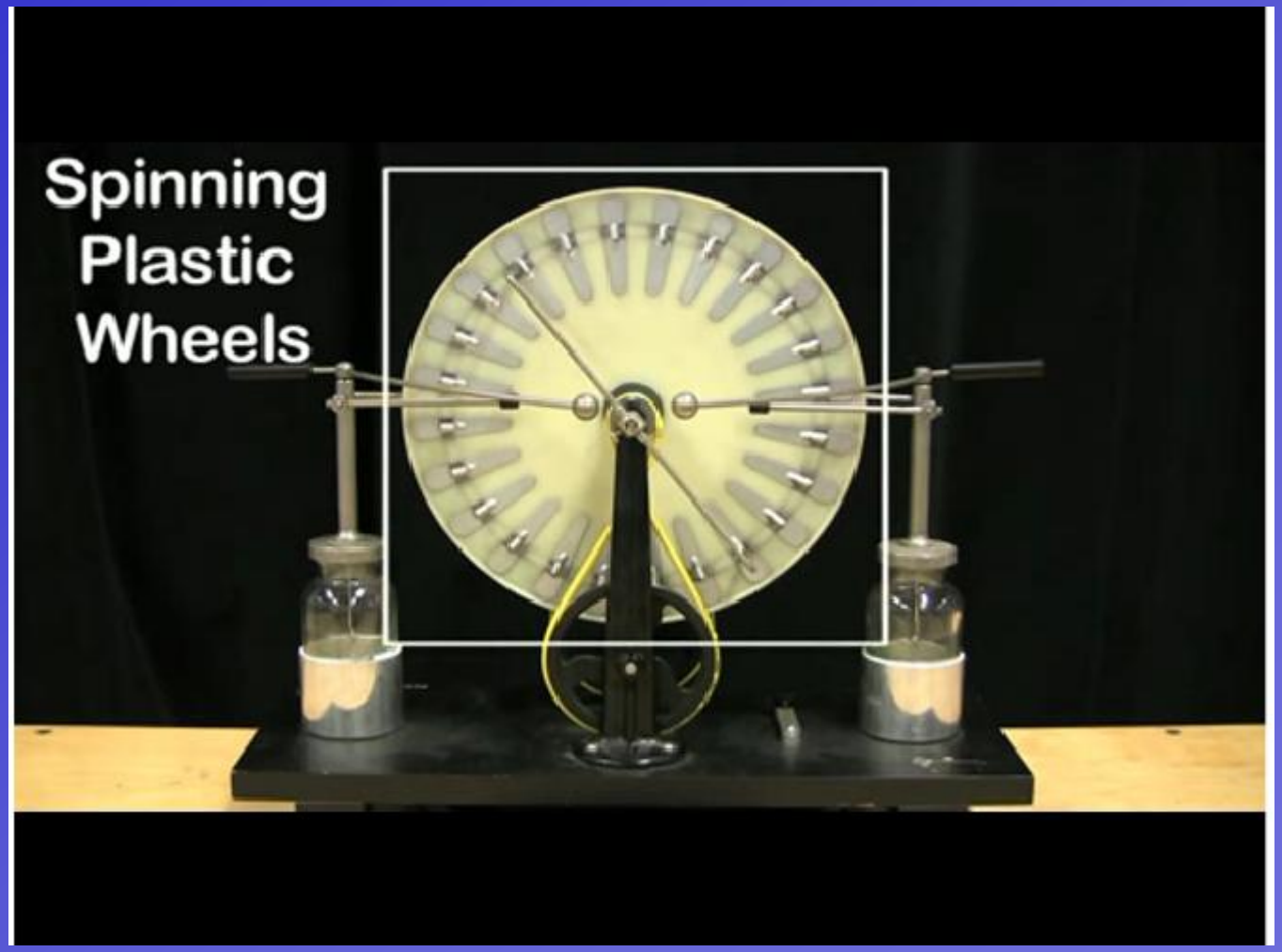


A plastic comb that has been charged by running it through your hair attracts neutral objects such as bits of paper or, as seen here, drops of water.

Table 8.1
The Electrostatic Series

Cat's fur	Low affinity for electrons
Acetate	
Glass	
Wool	
Lead	
Silk	
Wax	
Ebonite	
Copper	
Rubber	
Amber	
Sulphur	High affinity for electrons
Gold	

Charging by induction



Static electricity(静电学）

The property of electric charge:

1. There are two kinds of electric charge, positive charge and negative charge.

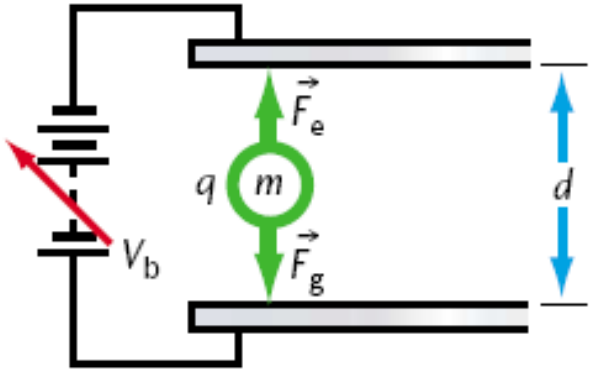
2. Like charges repel. And unlike charge attract.

3. Free charge is quantified in positive or negative integral multiples of e.

$$e=1.602*10^{-19}\ C$$

Fig.8.49 Millikan's oil-drop apparatus

Fig.8.50 Millikan used a horizontally oriented parallel-plate capacitor to create the electric force because of its uniform electric field and the ease of controlling and calculating the electric field strength



The conservation law of electric charge

The net charge is the same before and after any interaction

Coulomb’s law

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^3}\right)\vec{r}_{12}$$

Where ε_0 is the permittivity（介电系数，电容率） of free space

The application of static electric force: electrostatic copier

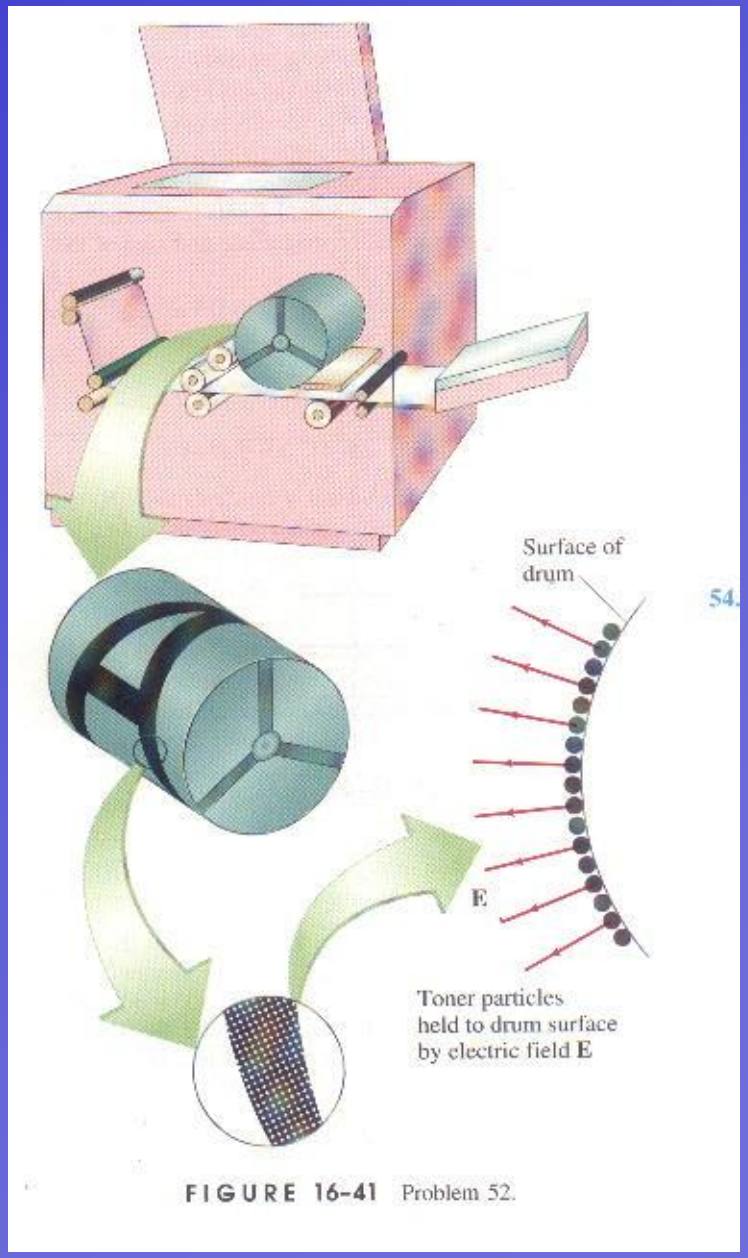
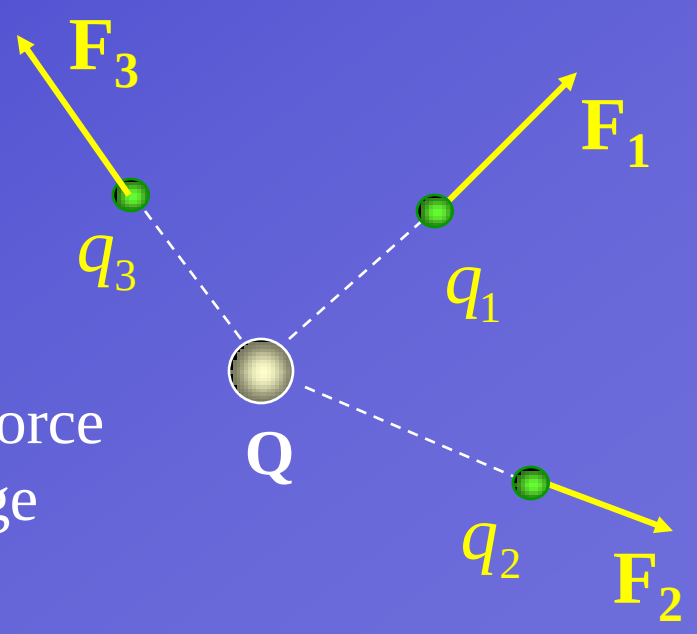


FIGURE 16-41 Problem 52.

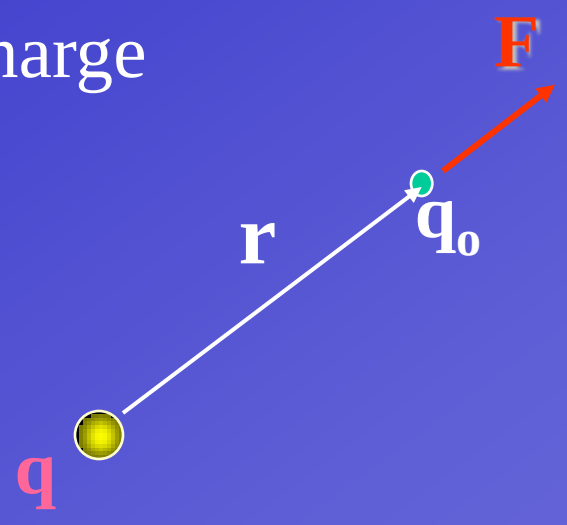
Electric field(电场）

$$\vec{E} = \frac{\vec{F}}{q_o}$$



The electric field is defined as the force on an infinitely small positive charge divided by the charge.

Electric field of a single charge



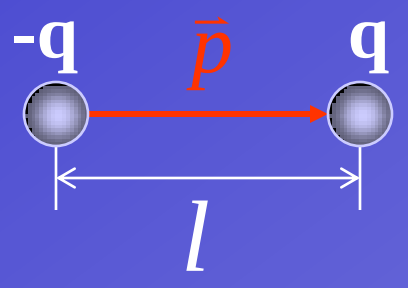
$$\vec{F} = \frac{q q_o}{4\pi\varepsilon_o r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_o}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_o r^2}\hat{r}$$

Electric dipole（电偶极子）:

Two charges with equal magnitude but opposite sign separated by l



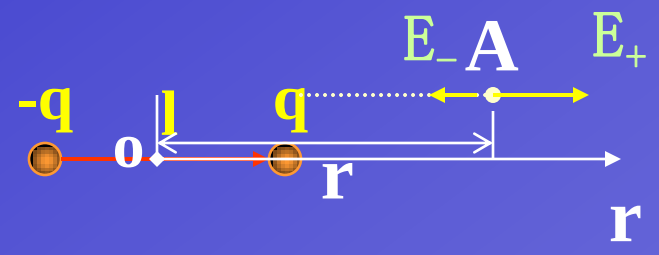
The Moment of electric dipole(电偶极矩）

$$\vec{p} = q\vec{l}$$

The electric field of the electric dipole

Along the line

$$E_+ = \frac{q}{4\pi\varepsilon_o (r - l/2)^2}$$



$$E_- = \frac{-q}{4\pi\varepsilon_o (r + l/2)^2}$$

$$E_A = E_+ + E_- = \frac{q}{4\pi\varepsilon_o}\frac{2rl}{r^4}\frac{1}{\left(1 - l^2/4r^2\right)^2}$$

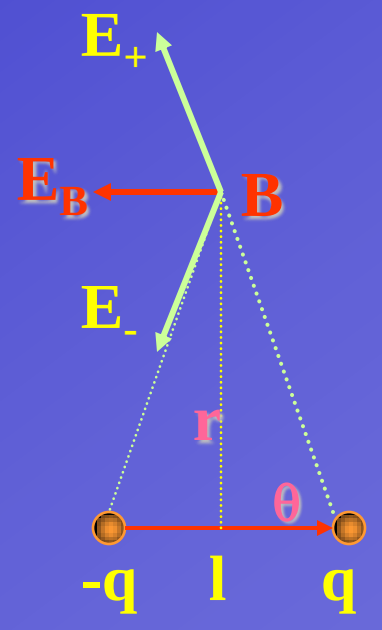
$$\because \quad r >> l$$

$$\therefore \quad l^2/4r^2 \approx 0$$

$$E_A \approx \frac{2ql}{4\pi\varepsilon_o r^3} = \frac{2p}{4\pi\varepsilon_o r^3}$$

Along the perpendicular axis that bisects the line between these two charges

$$E_B = E_+ \cos\theta + E_- \cos\theta$$



$$E_+ = E_- = \frac{q}{4\pi\varepsilon_o\left(r^2 + l^2/4\right)^2}$$

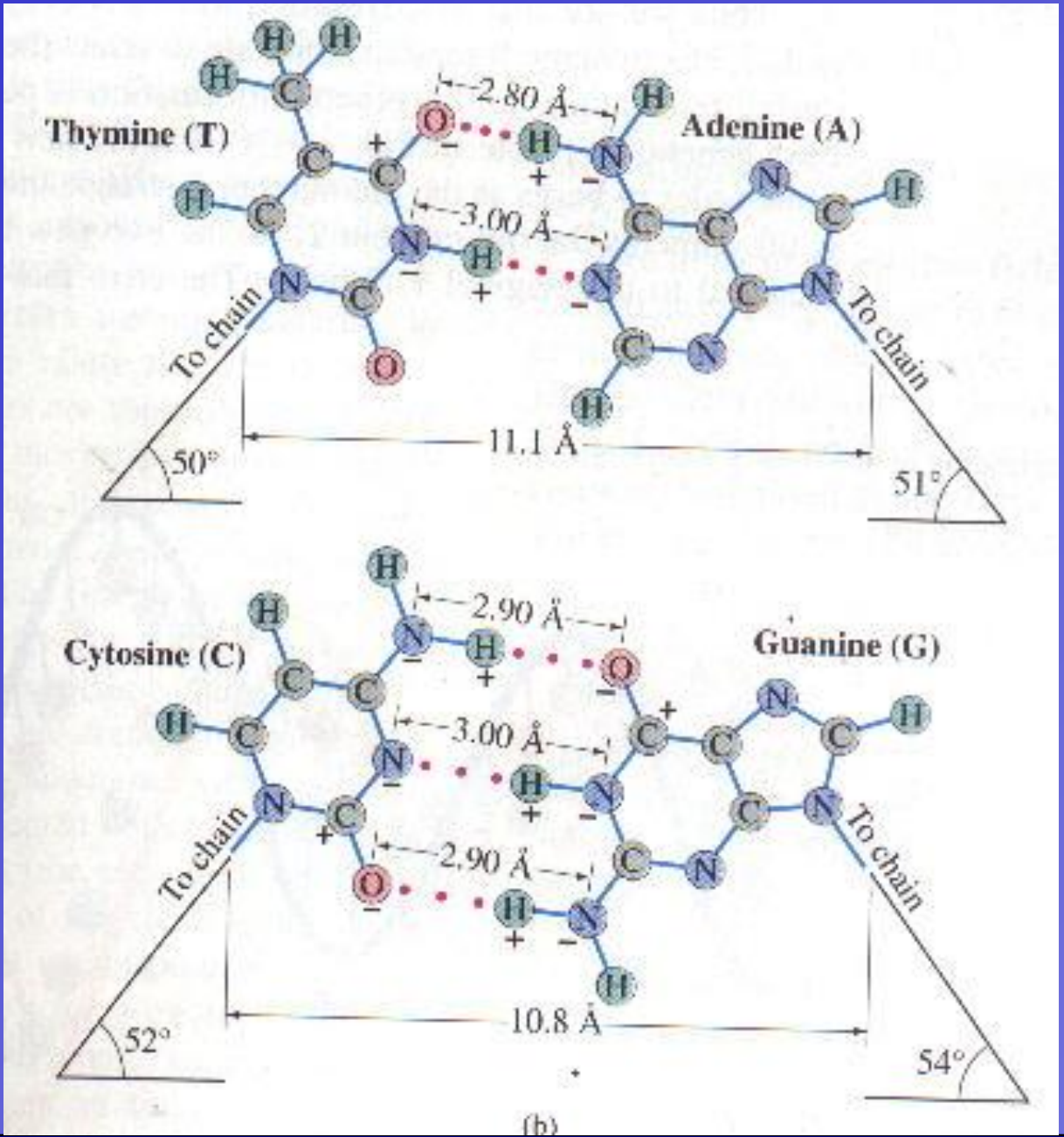
$$\cos\theta = \frac{l}{2\sqrt{r^2 + l^2/4}}$$

$$E_B = 2E_+ \cos\theta = \frac{ql}{4\pi\varepsilon_o\left(r^2 + l^2/4\right)^{3/2}}$$

r>>l

$$E_B = \frac{ql}{4\pi\varepsilon_o r^3} = -\frac{p}{4\pi\varepsilon_o r^3}$$

The electric dipole in DNA



Electric field of charge continuously distributed

The superposition law for electric field

$$\mathbf{E}=\int d\mathbf{E}=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\mathbf{r}$$

For a charged line segment：

线电荷： $dq=\lambda dl$

Surface：

面电荷： $dq=\sigma ds$

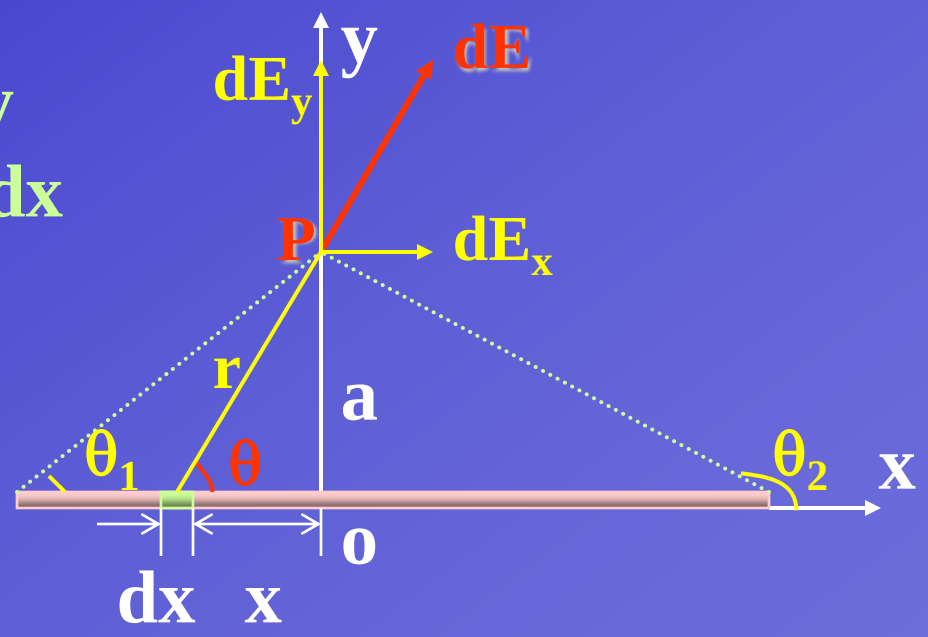
Volume：

体电荷 $dq=\rho dV$

：

A straight insulating rod of length L carries a uniform linear charge Q。 Determine the electric field at point P， a distance **a** from the rod along the line perpendicular to the rod.

solution : **For elementary charge： dq=λdx**



$$dE = \frac{\lambda\, dx}{4\pi\varepsilon_o r^2}$$

$$dE_x = dE\cos\theta$$

$$= \frac{\lambda\, dx\cos\theta}{4\pi\varepsilon_o r^2}$$

$$dE_y = dE\sin\theta = \frac{\lambda\, dx\sin\theta}{4\pi\varepsilon_o r^2}$$

$$dE_x = \frac{\lambda\, dx \cos\theta}{4\pi\varepsilon_o r^2} \qquad dE_y = \frac{\lambda\, dx \sin\theta}{4\pi\varepsilon_o r^2}$$

$$r = \frac{a}{\sin\theta} = a\csc\theta \qquad x = -a\,ctg\theta$$

$$dx = a\csc^2\theta\, d\theta$$

$$dE_x = \frac{\lambda dx\cos\theta}{4\pi\varepsilon_o r^2} = \frac{\lambda\, a\csc^2\theta\cos\theta\, d\theta}{4\pi\varepsilon_o a^2\csc^2\theta} = \frac{\lambda\cos\theta}{4\pi\varepsilon_o a}\, d\theta$$

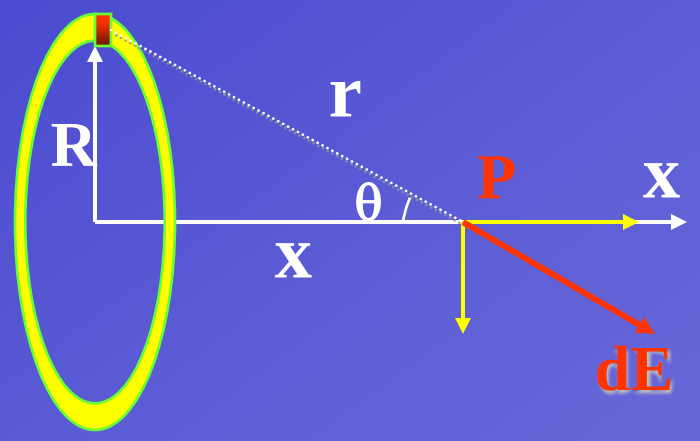
$$E_x = \int \frac{\lambda \cos\theta}{4\pi\varepsilon_o a} d\theta = \frac{\lambda}{4\pi\varepsilon_o a}\left(\sin\theta_2 - \sin\theta_1\right)$$

$$dE_y = \frac{\lambda \sin\theta}{4\pi\varepsilon_o a} d\theta \quad E_y = \int dE_y = \frac{\lambda}{4\pi\varepsilon_o a}\left(\cos\theta_1 - \cos\theta_2\right)$$

For infinite long line： $\theta_1 = 0$ ， $\theta_2 = \pi$

$$E_x = 0 \qquad E = E_y = \frac{\lambda}{2\pi\varepsilon_o a}$$

Charge q distributed along a ring of radius R uniformly, find the distribution of electric field along its axis



$$dq = \frac{q}{2\pi R} dl$$

$$dE = \frac{dq}{4\pi \varepsilon_o r^2} = \frac{q dl}{8\pi^2 R \varepsilon_o r^2}$$

$$E = E_x = \int_L dE_x = \int_L dE \cos\theta = \int_L \frac{x}{r} \cdot dE$$

$$E = \int_0^{2\pi R} \frac{q x dl}{8\pi^2 \varepsilon_o R r^3} = \frac{qx}{4\pi\varepsilon_o \left(x^2 + R^2\right)^{3/2}}$$