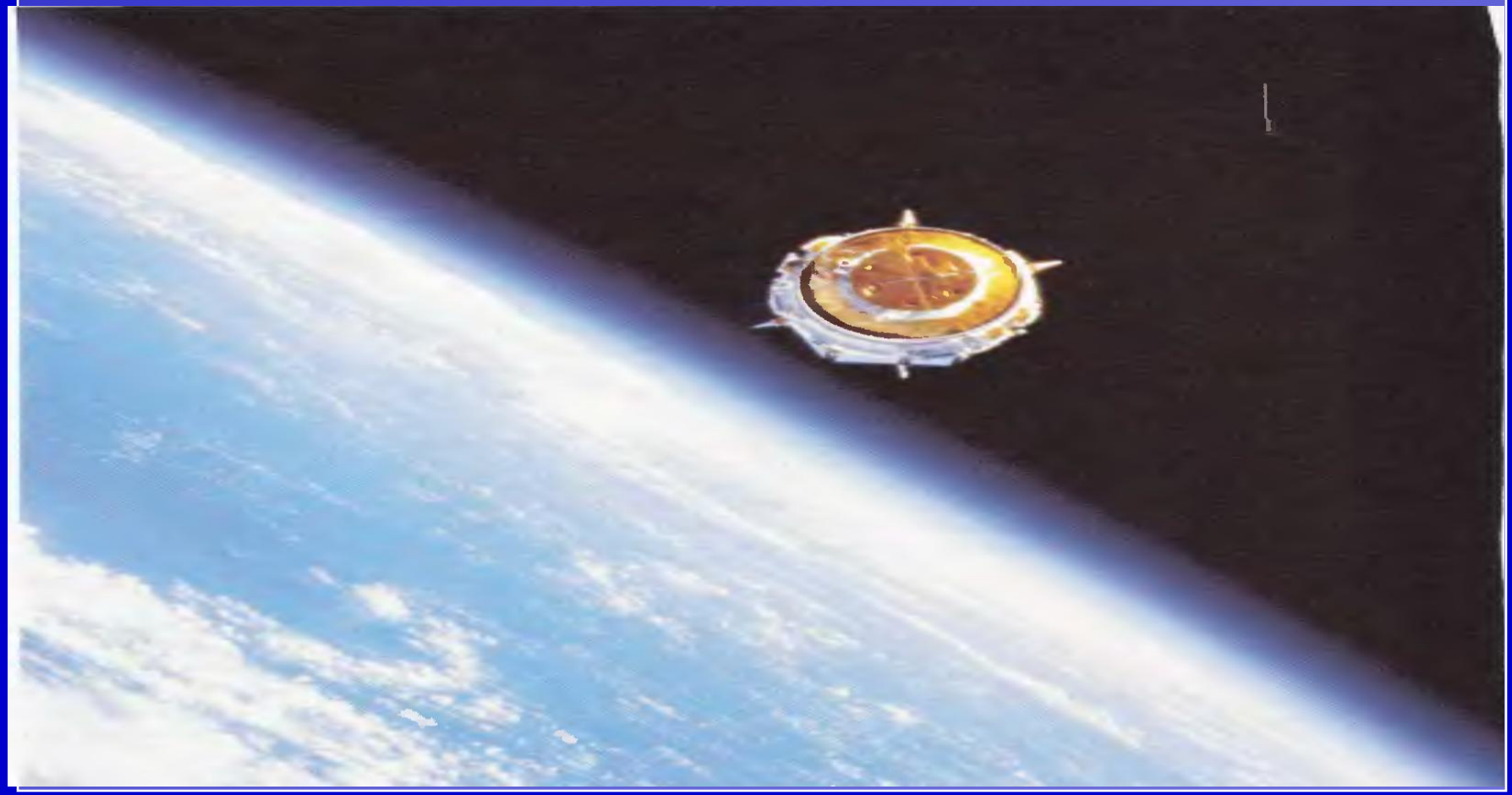
# Gravity



# Agenda Today

1 Kepler's Laws

2 Newton's Law of gravity

3 gravitational field and potential energy

4 Einstein's theory of gravity

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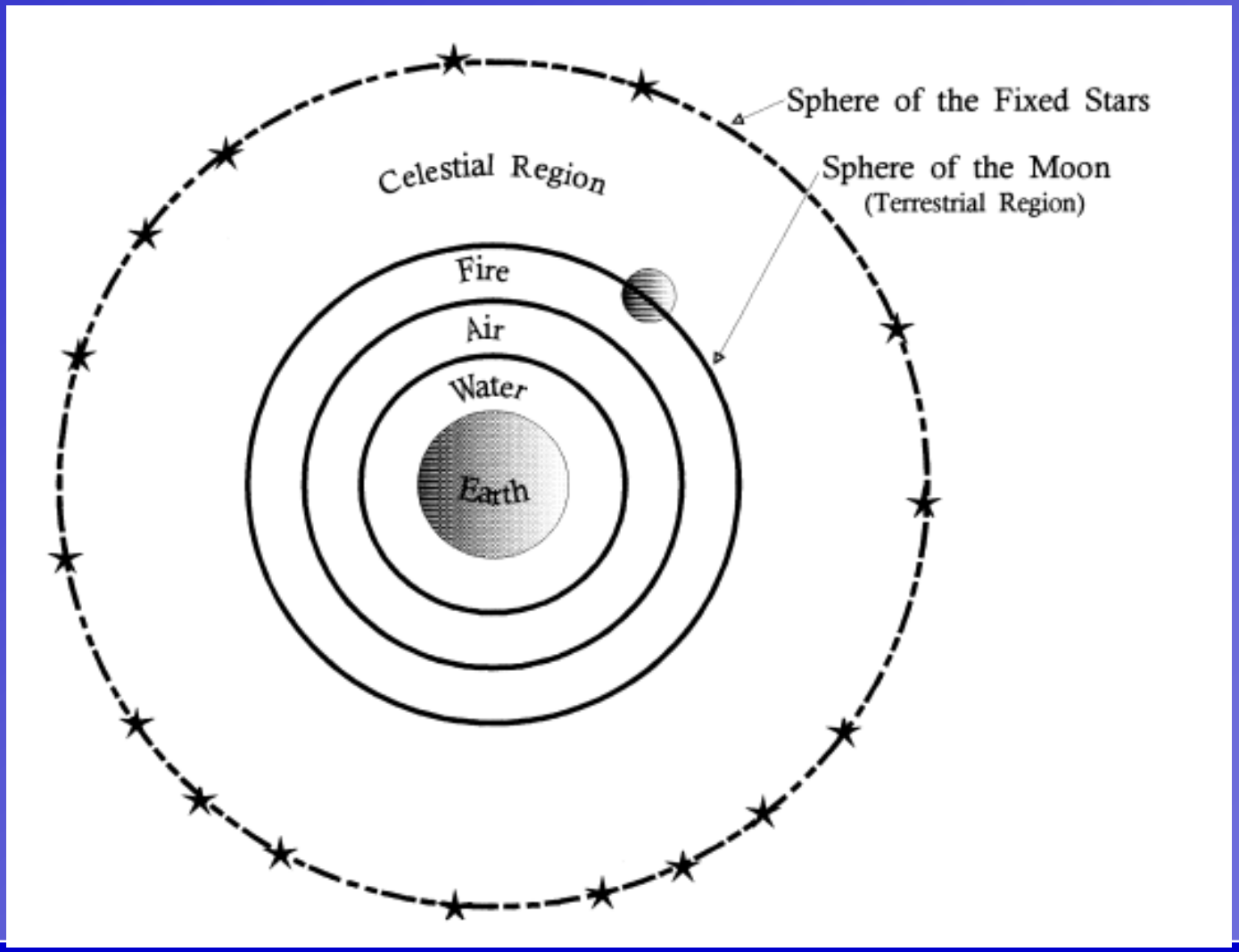
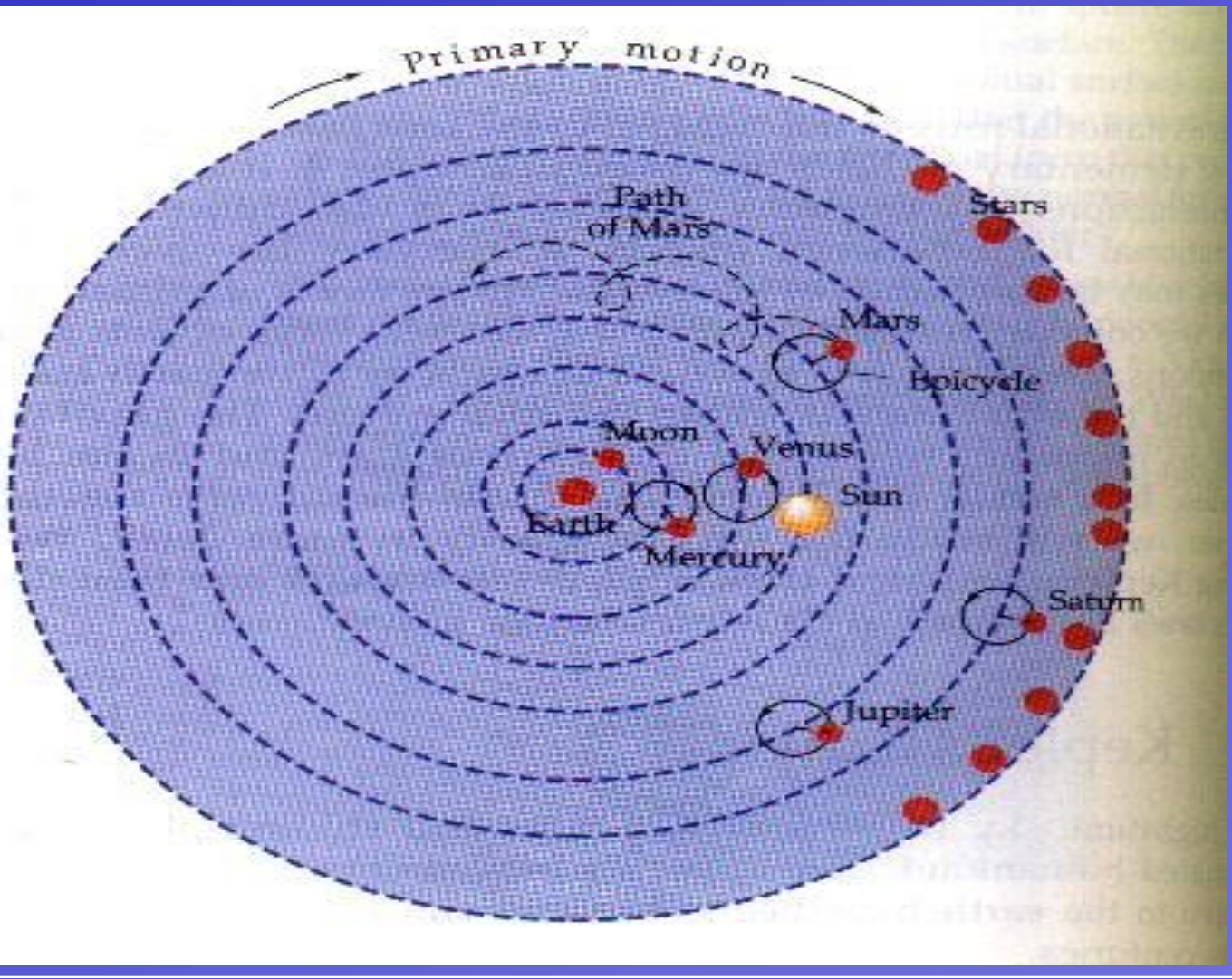


# Ptolemy's model of universe



# The heroes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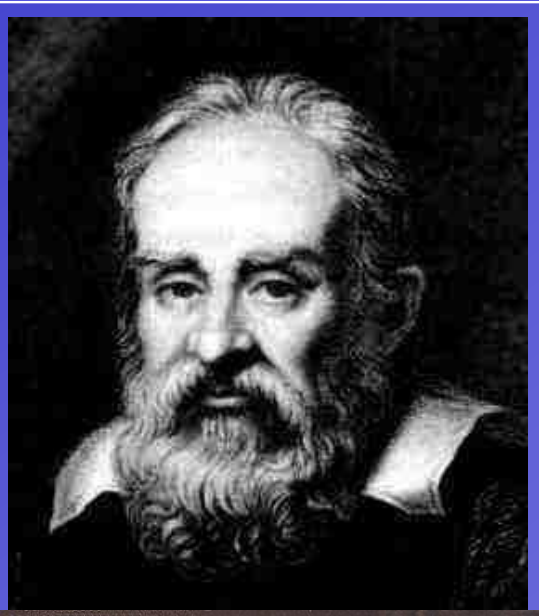

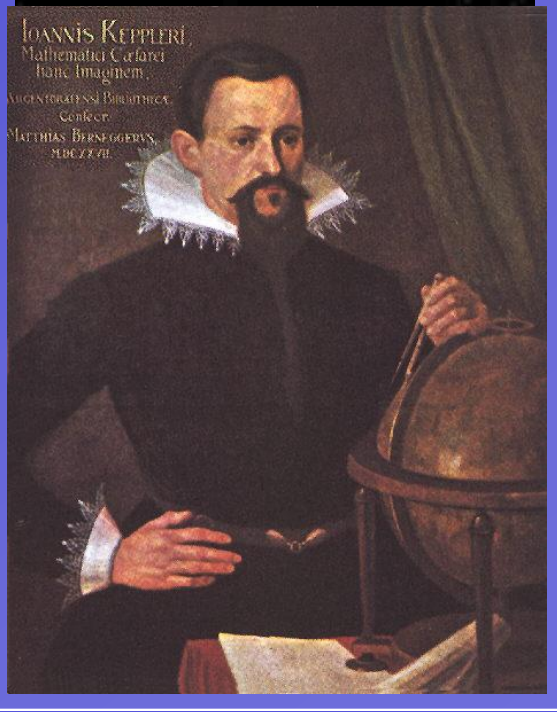
IOANNIS KEPPLERI
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# Kepler's laws

The law of orbit: All planets move in elliptical with the sun at one focus

The Law of area: A line joining any planet to the sun sweeps out equal areas in equal times.

Angular momentum（角动量）:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

The law of period:

the square of the period of any planet is proportional to the cube of the planet's mean distance from the sun.

$$\frac{T^2}{R^3} = C$$

Newton's Law of gravity:

$$F_{12} = \frac{G m_1 m_2}{r_{12}^2} \hat{r}_{12}$$

G: gravitational constant

For any planet,

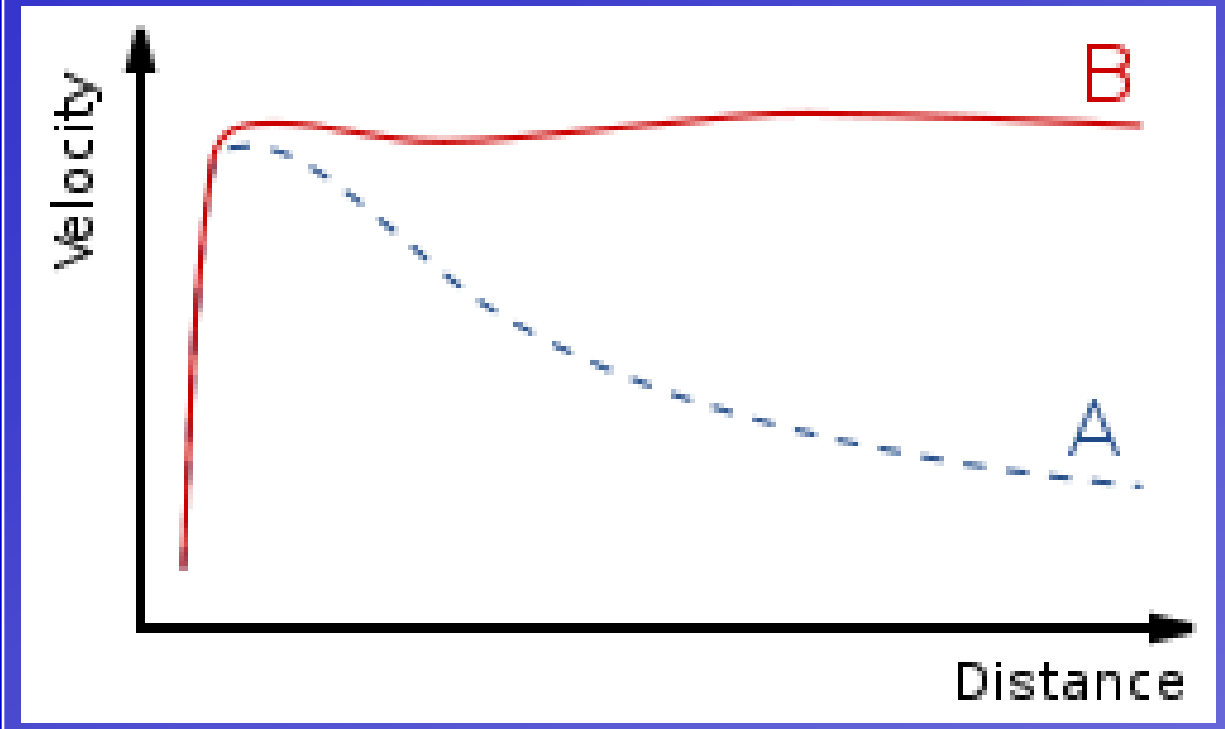
$$\frac{GM_s M_p}{r^2} = M_p \frac{v^2}{r}$$

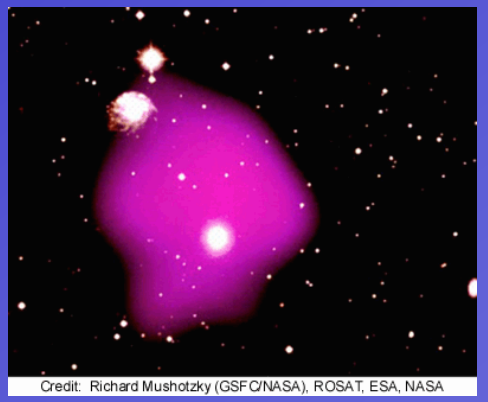
$$v = \frac{2\pi r}{T}$$

We have:

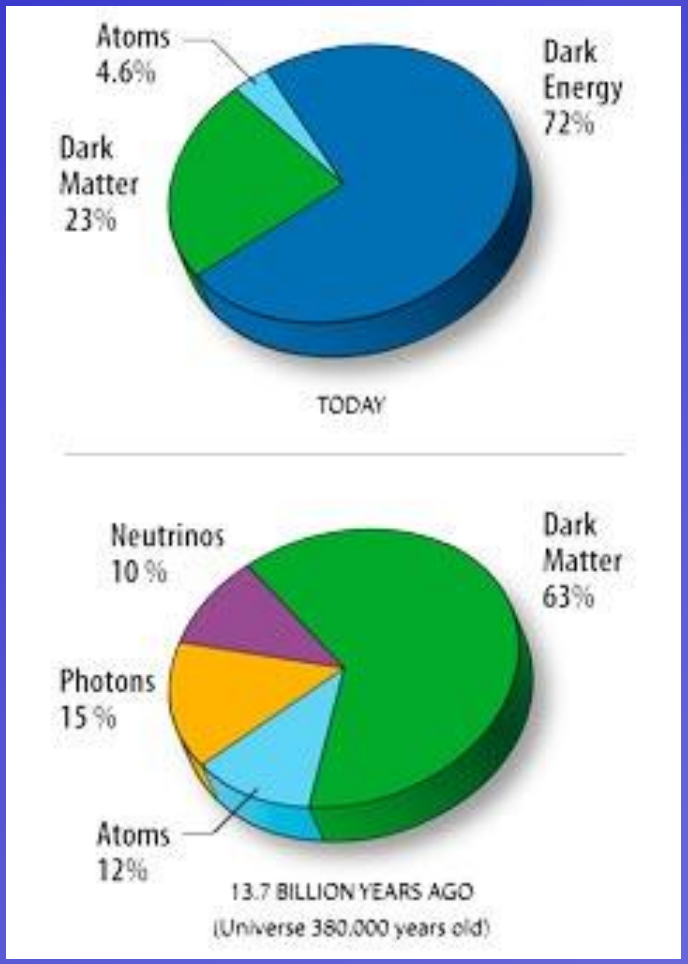
$$T^2 = \frac{4\pi^2}{GM_s} r^3$$

Kepler's law of period





Credit: Richard Mushotzky (GSFC/NASA), ROSAT, ESA, NASA

# The existence of dark matter




And dark energy…….

# Gravity near the earth surface

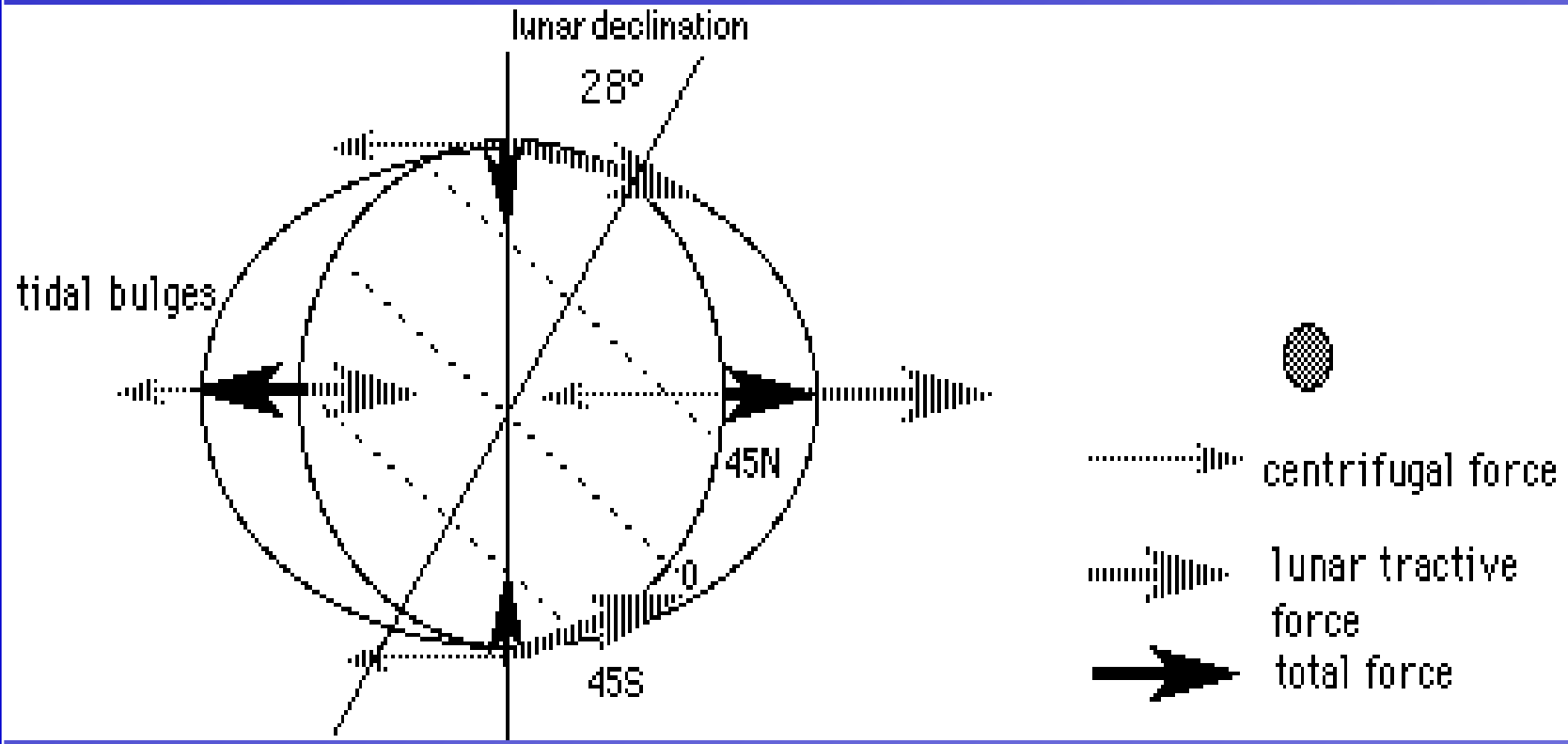
Measure gravity at different latitude

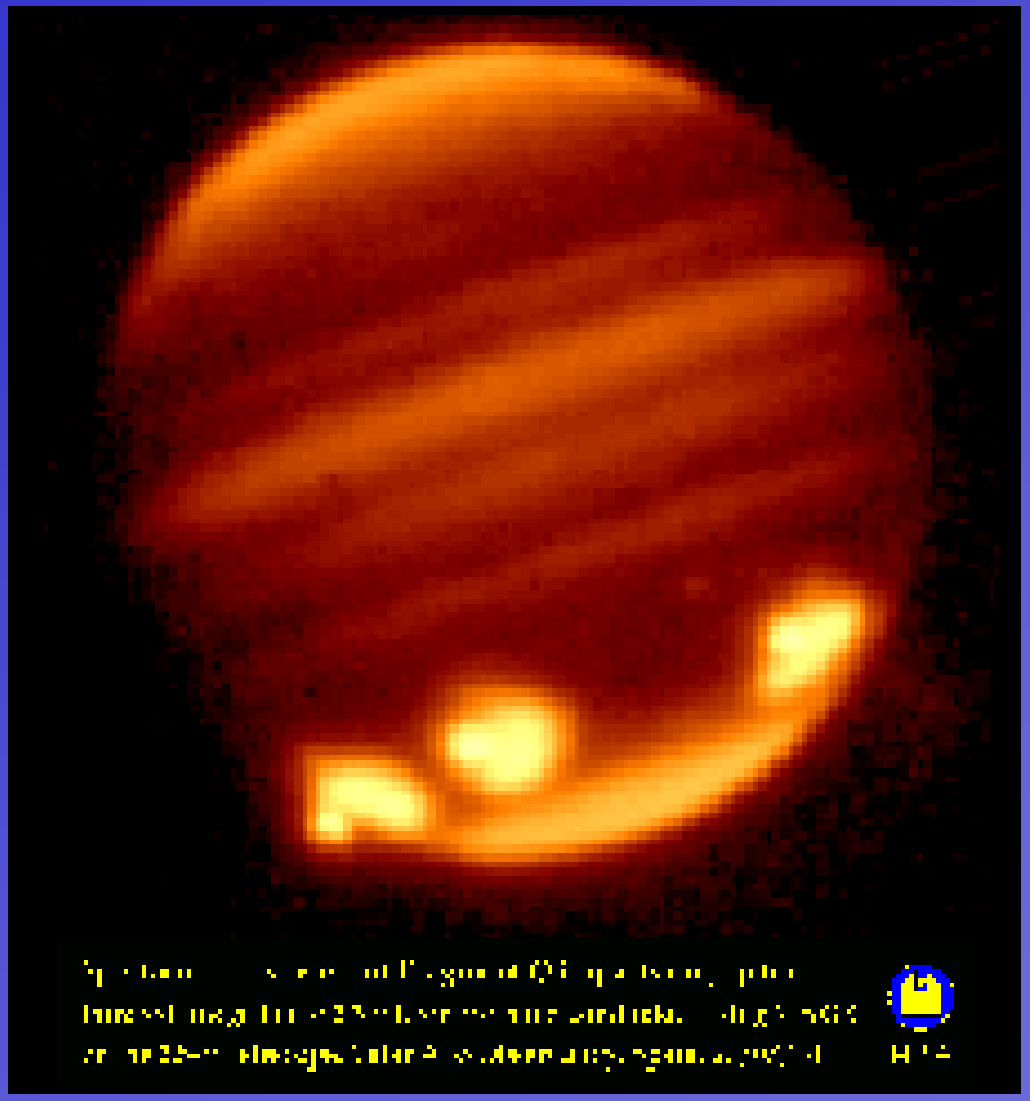
$$m\vec{g}' = m\vec{g} - \vec{F}_c$$

Measure gravity at different altitude

$$m\vec{g}' = \frac{m\vec{g}}{(1+\frac{h}{R_e})^2} \cong m\vec{g}(1-\frac{2h}{R_e})$$

# Tide force


lunar declination
28°
tidal bulges
45N
0
45S
centrifugal force
lunar tractive force
total force



The great Tide force split up the comet Shoemaker-levy 9

Gravitational field :

$$g = -G\frac{M}{r_{12}^2}\hat{r}_{12}$$

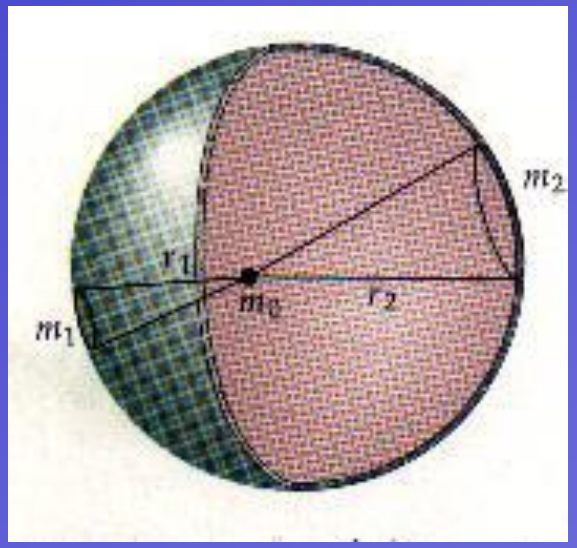
The principle of superposition(叠加原理）:

$$g_{net} = g + g + \cdots g_n$$

# The Gravitational field of a spherical shell :

Inside the sphere:

$$g = 0$$



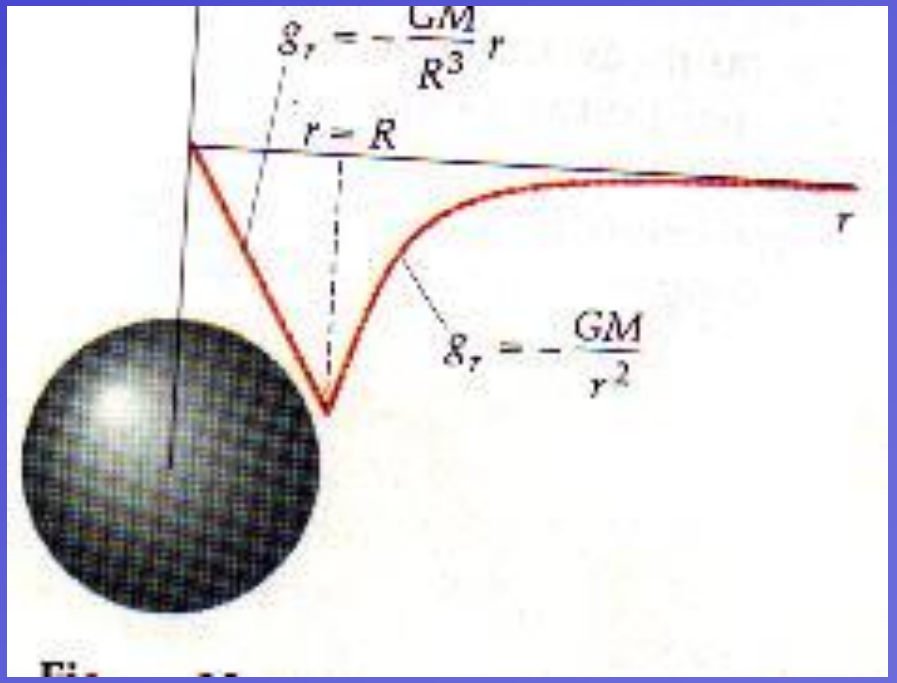

Outside the sphere:

$$g = -\frac{GM}{r_{12}^2}\hat{r}_{12}$$

# The gravitational field of a solid sphere:

$$g = -\frac{GM}{R^3} r\hat{\mathbf{r}} \quad r < R$$

$$g = -\frac{GM}{r^2}\hat{\mathbf{r}} \quad r > R$$



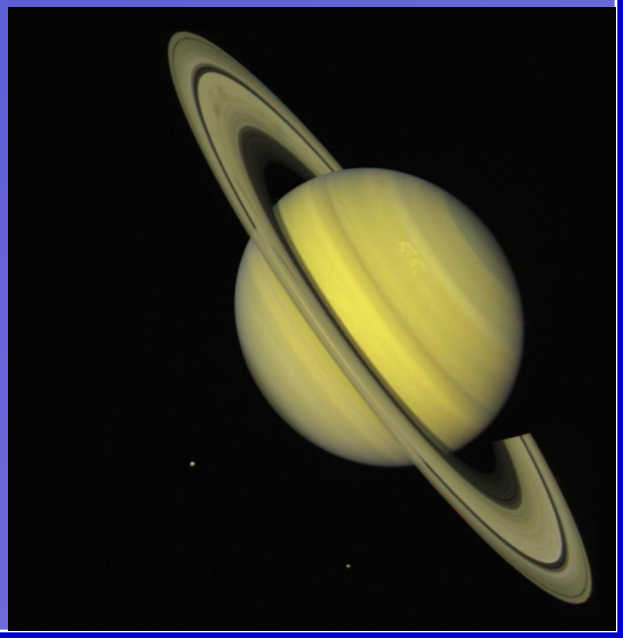
# Gravitational potential Energy

$$U = -\frac{GMm}{r}$$

Choose infinite as the reference point of zero potential energy

If we choose the ground as the reference point:

Quiz: planets such as Saturn posses nearly circular rings, consider a homogeneous ring of mass M and radius R. Suppose a particle with mass m is moving along the axis with initial speed v toward the center, and its initial position is x from the center, find the speed of particle when it is passing through the center of the ring.



# Satellite: orbit and energy

$$U = -\frac{GMm}{r}$$

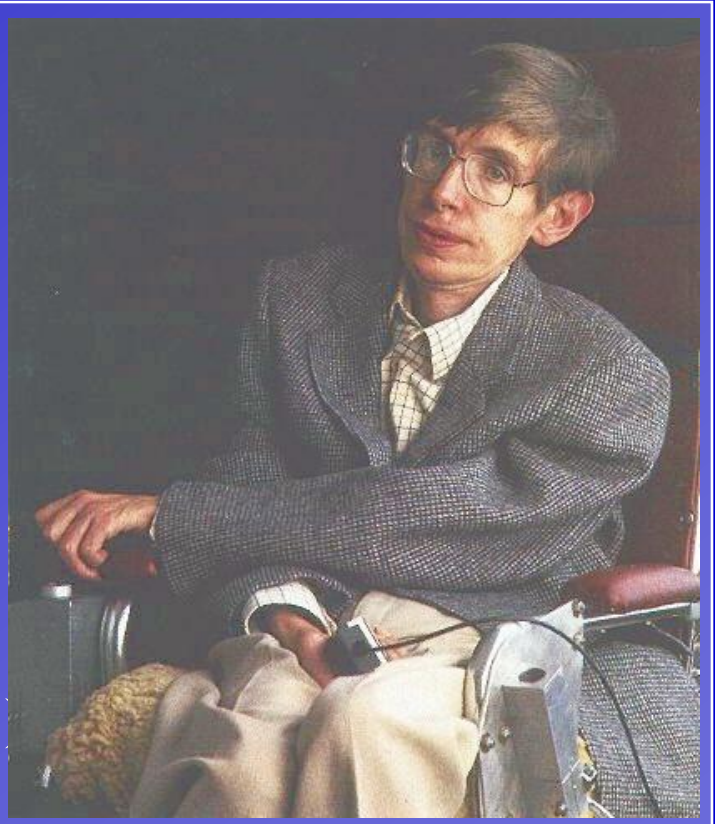
$$E_K = -\frac{U}{2}$$

$$E = E_k + U = -K \quad (circular \quad orbit)$$

$$E = -\frac{GMm}{2a} \quad (elliptical \quad orbit)$$

Escape speed（逃逸速度）：

$$v = \sqrt{\frac{2GM}{R}}$$



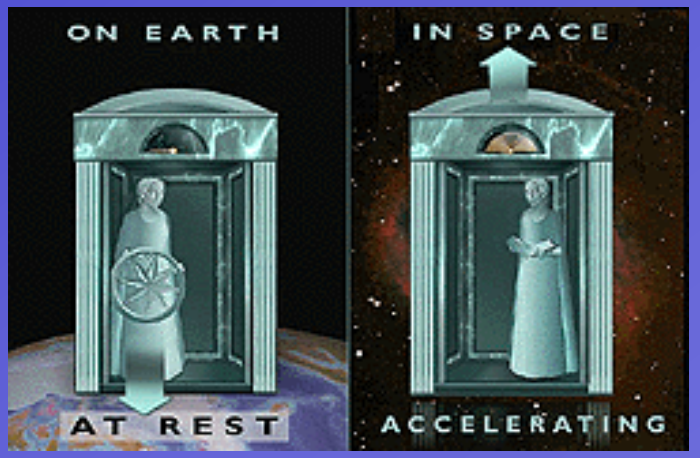
Schwarzschild radius（史瓦西半径）

$$R_s = \frac{2GM}{c^2}$$

Einstein ‘s theory of gravity:

The principle of equivalence:

（等效原理）




A homogeneous gravitational field is completely equivalent to a uniformly accelerated reference frame

Consequences of Einstein's theory

Gravitational time dilation and frequency shift

Light deflection