

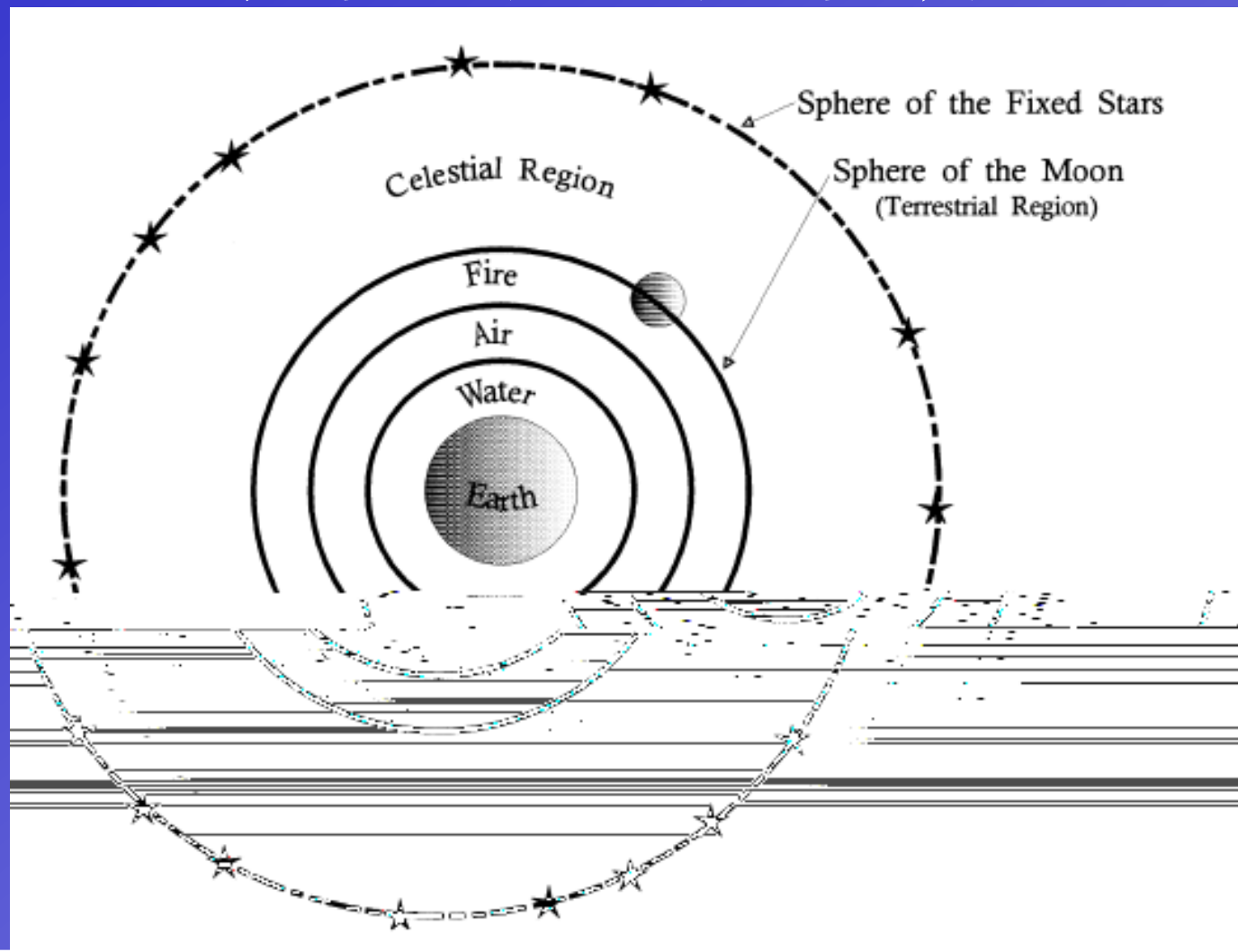
Gravity



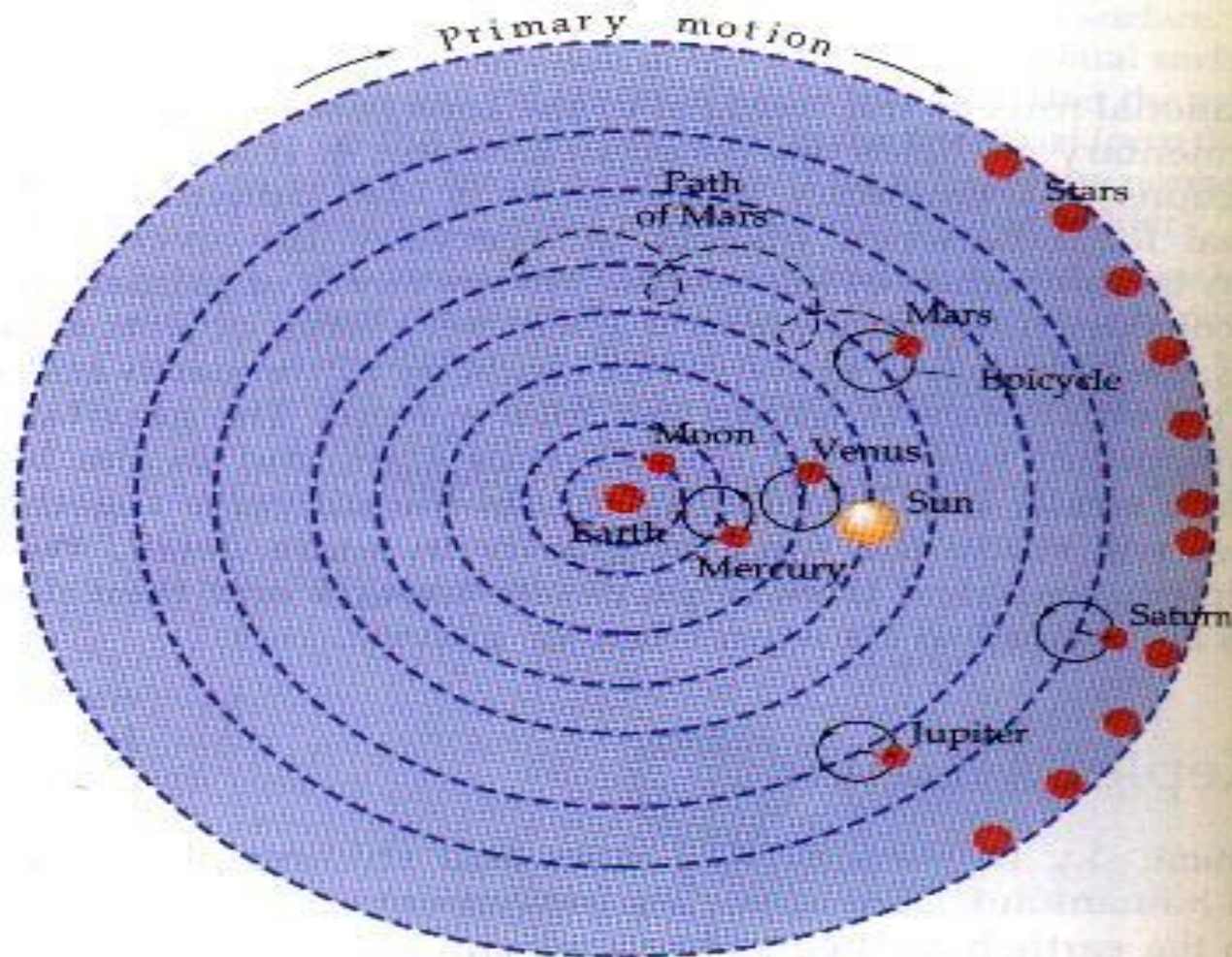
Agenda Today

- 1 Kepler's Laws
- 2 Newton's Law of gravity
- 3 gravitational field and potential energy
- 4 Einstein's theory of gravity

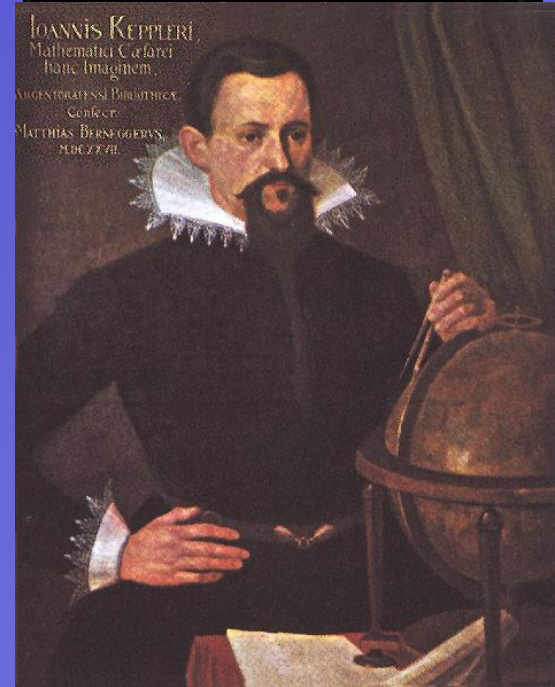
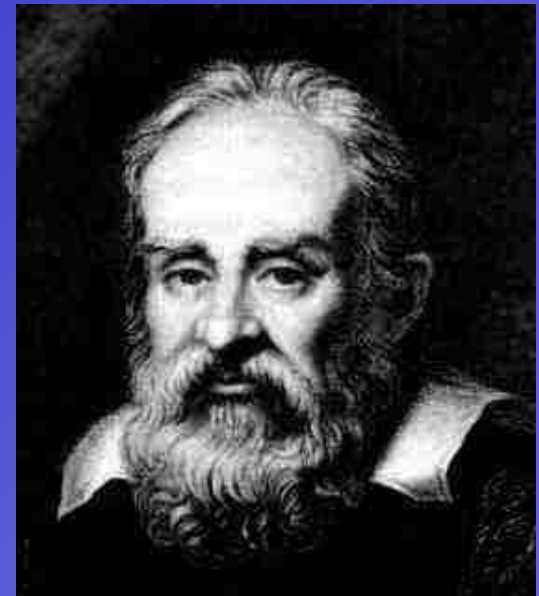
天何所沓？十二焉分？
日月安属？列星安陈？



Ptolemy's model of universe



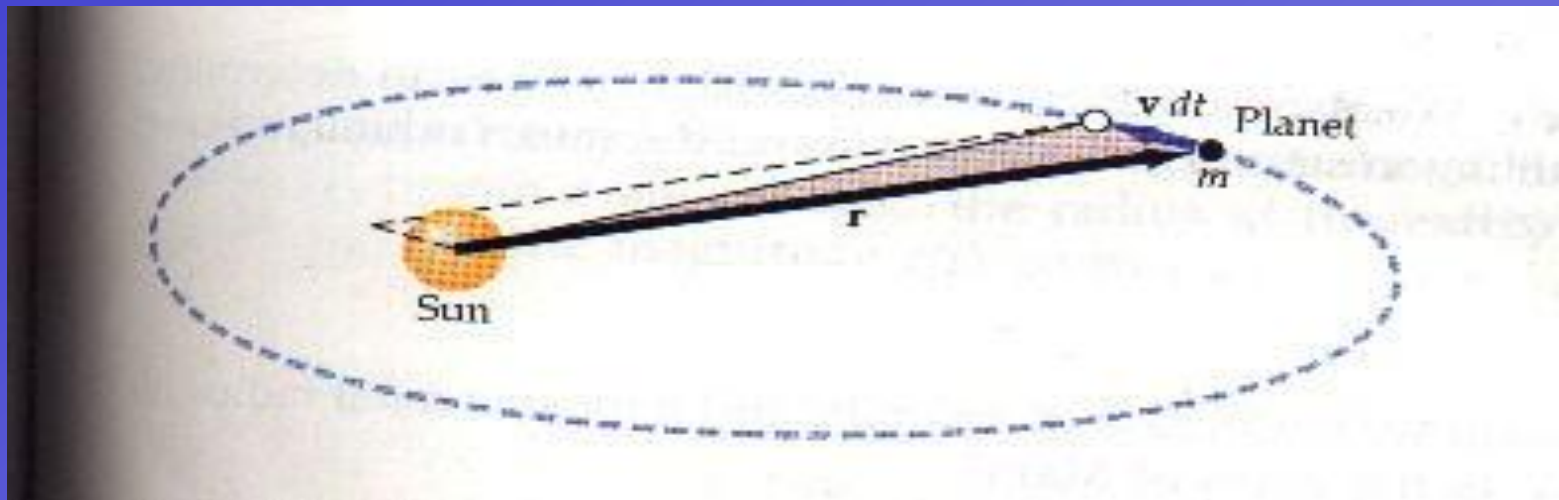
The heroes



Kepler's laws

The law of orbit: All planets move in elliptical with the sun at one focus

The Law of area: A line joining any planet to the sun sweeps out equal areas in equal times.



Angular momentum (角动量) :

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

The law of period:

the square of the period of any planet is proportional to the cube of the planet's mean distance from the sun.

$$\frac{T^2}{R^3} = C$$

Newton's Law of gravity:

$$\mathbf{F}_{12} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$$



G: gravitational constant

For any planet,

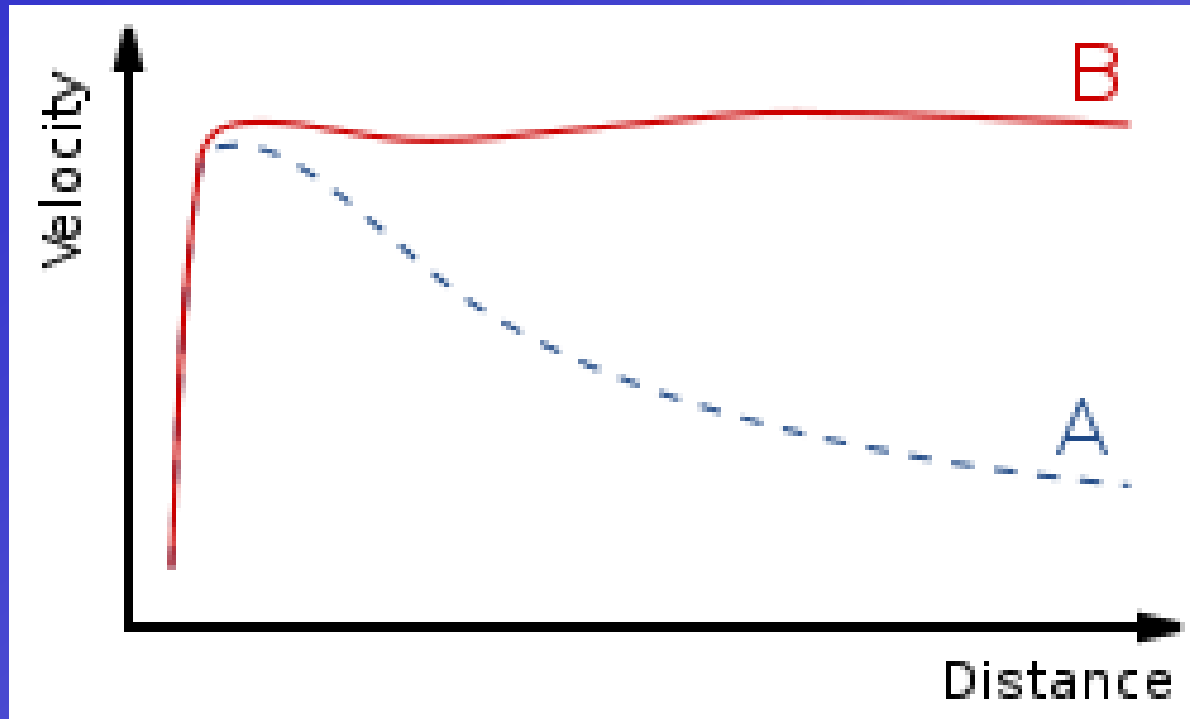
$$\frac{GM_s M_p}{r^2} = M_p \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

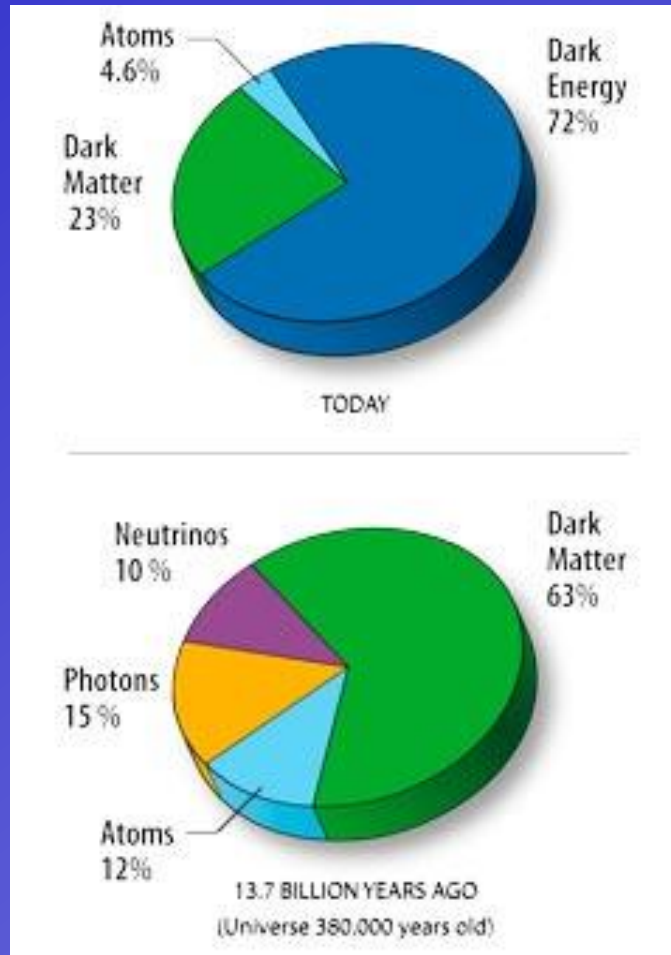
We have:

$$T^2 = \frac{4\pi^2}{GM_s} r^3$$

Kepler's law of
period



The existence of dark matter



And dark energy.....

Gravity near the earth surface

Measure gravity at different latitude

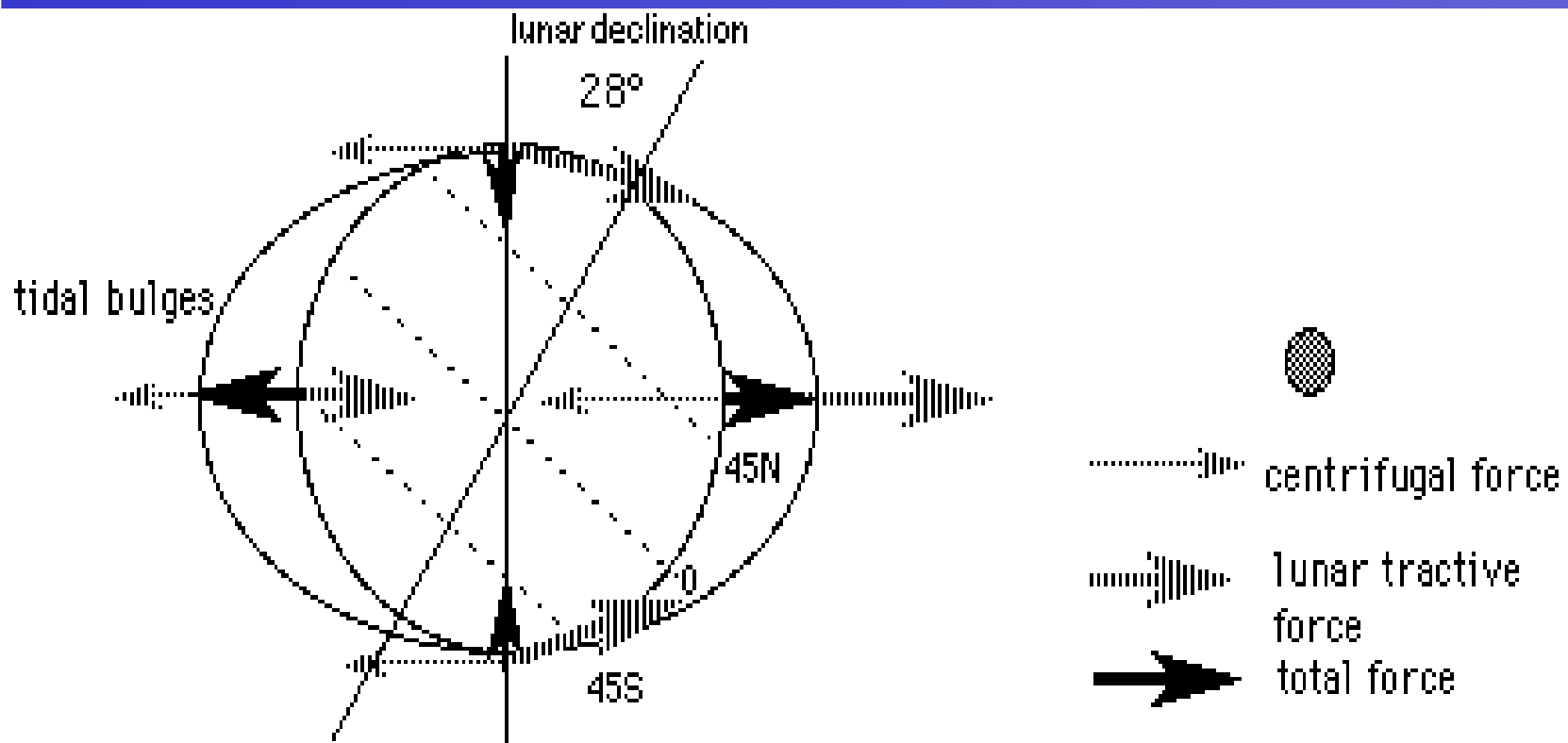
$$m\vec{g}' = m\vec{g} - \vec{F}_c$$

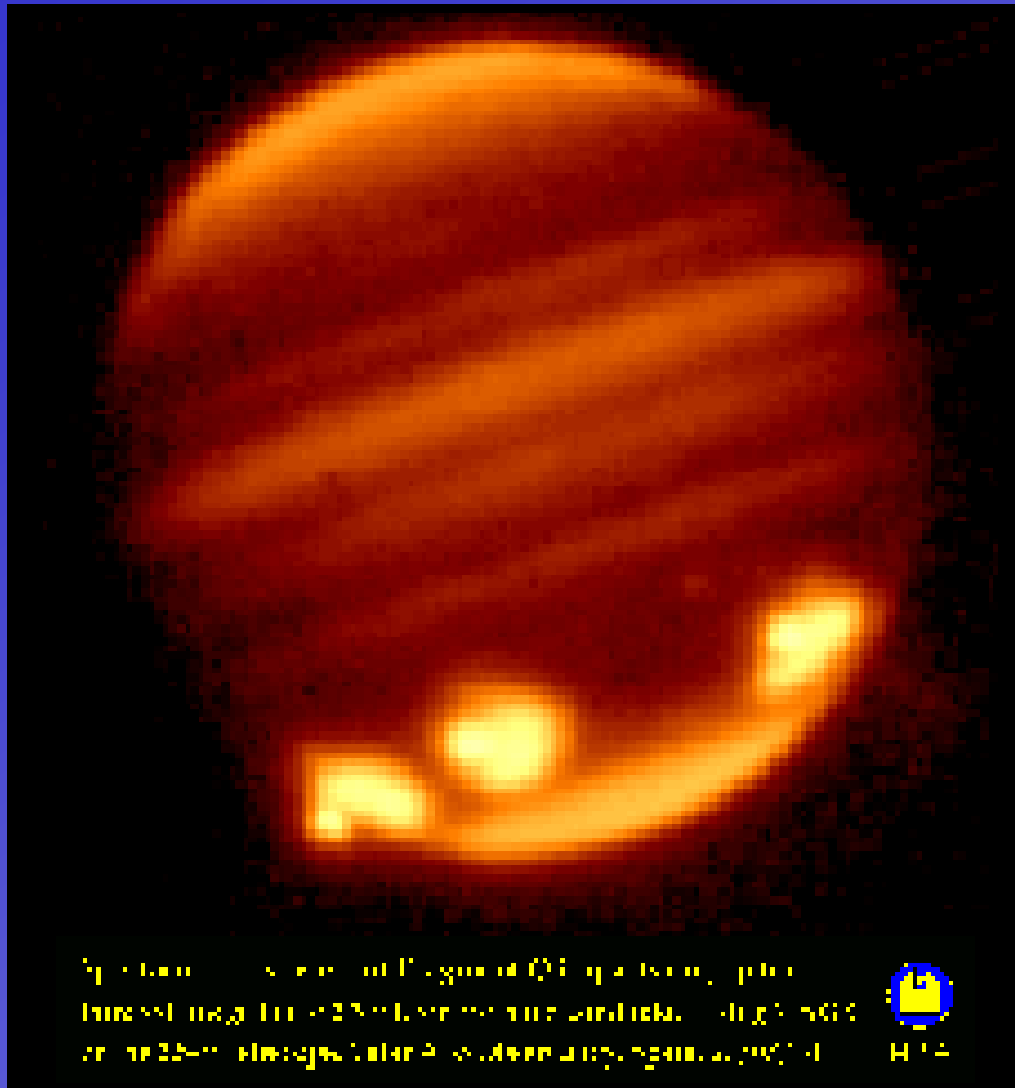
Measure gravity at different altitude

$$m\vec{g}' = \frac{m\vec{g}}{\left(1 + \frac{h}{R_e}\right)^2} \cong m\vec{g} \left(1 - \frac{2h}{R_e}\right)$$



Tide force





Spitzer Science Center, Division of Geological and Planetary Sciences
Imaging and Spectroscopy Laboratory, NASA/JPL
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NASA

The great Tide force split up the comet Shoemaker-levy 9

Gravitational field :

$$g = -G \frac{M}{r_{12}^2} \hat{r}_{12}$$

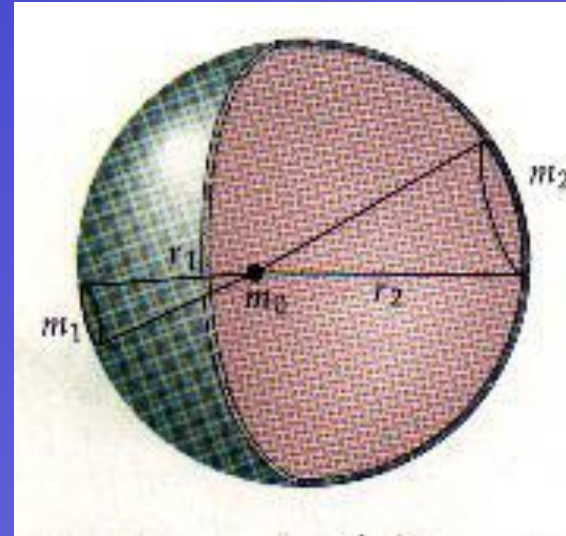
The principle of superposition(叠加原理) :

$$g_{net} = g_1 + g_2 + \dots + g_n$$

The Gravitational field of a spherical shell :

Inside the sphere:

$$g = 0$$



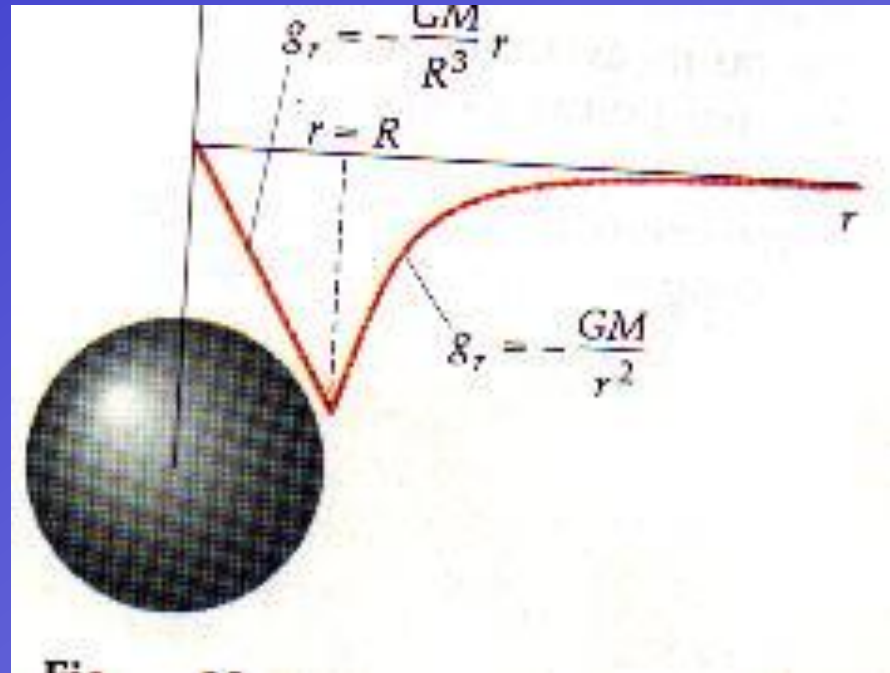
Outside the sphere:

$$g = -\frac{GM}{r_{12}^2} \hat{r}_{12}$$

The gravitational field of a solid sphere:

$$\mathbf{g} = -\frac{GM}{R^3} r \hat{\mathbf{r}} \quad r < R$$

$$\mathbf{g} = -\frac{GM}{r^2} \hat{\mathbf{r}} \quad r > R$$



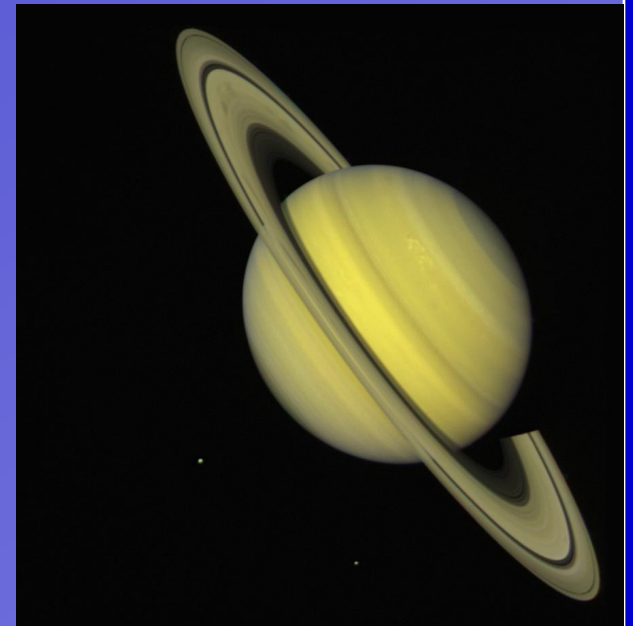
Gravitational potential Energy

$$U = -\frac{GMm}{r}$$

Choose infinite as the reference point of zero potential energy

If we choose the ground as the reference point:

Quiz: planets such as Saturn possess nearly circular rings, consider a homogeneous ring of mass M and radius R . Suppose a particle with mass m is moving along the axis with initial speed v toward the center, and its initial position is x from the center, find the speed of particle when it is passing through the center of the ring.



Satellite: orbit and energy

$$U = -\frac{GMm}{r}$$

$$E_K = -\frac{U}{2}$$

$$E = E_k + U = -K \quad (\text{circular orbit})$$

$$E = -\frac{GMm}{2a} \quad (\text{elliptical orbit})$$

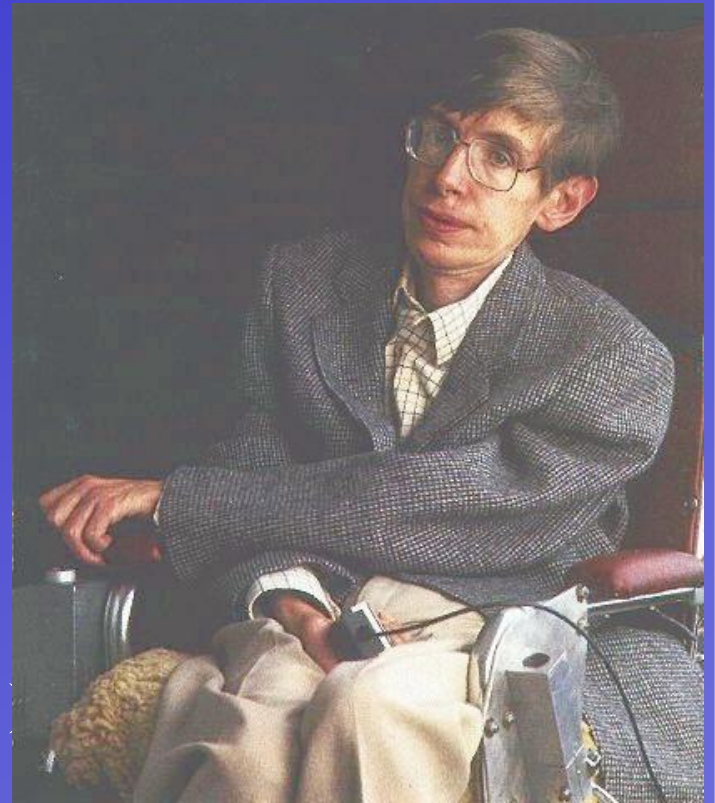


Escape speed (逃逸速度) :

$$v = \sqrt{\frac{2GM}{R}}$$

Schwarzschild radius (史瓦西半径)

$$R_s = \frac{2GM}{c^2}$$



Einstein 's theory of gravity:

The principle of equivalence:

(等效原理)

A homogeneous gravitational field is completely equivalent to a uniformly accelerated reference frame



Consequences of Einstein's theory

Gravitational time dilation and frequency shift

Light deflection

