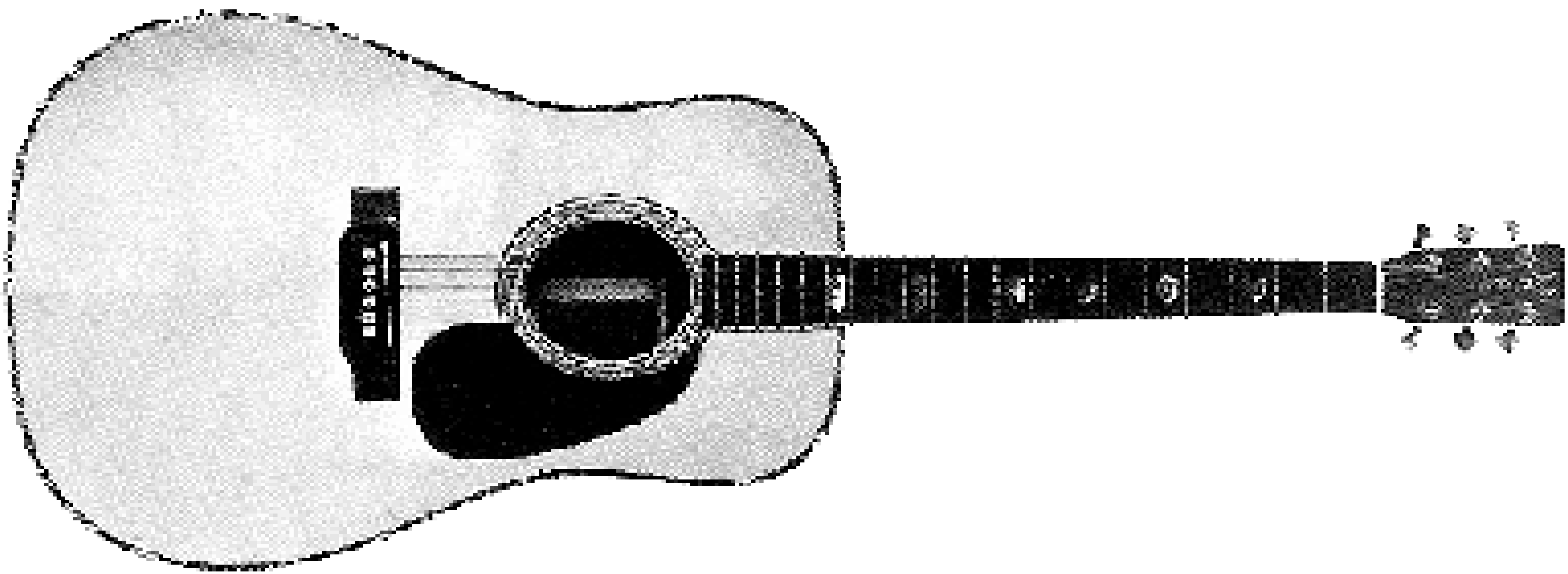
Mechanical waves (II)



Agenda today

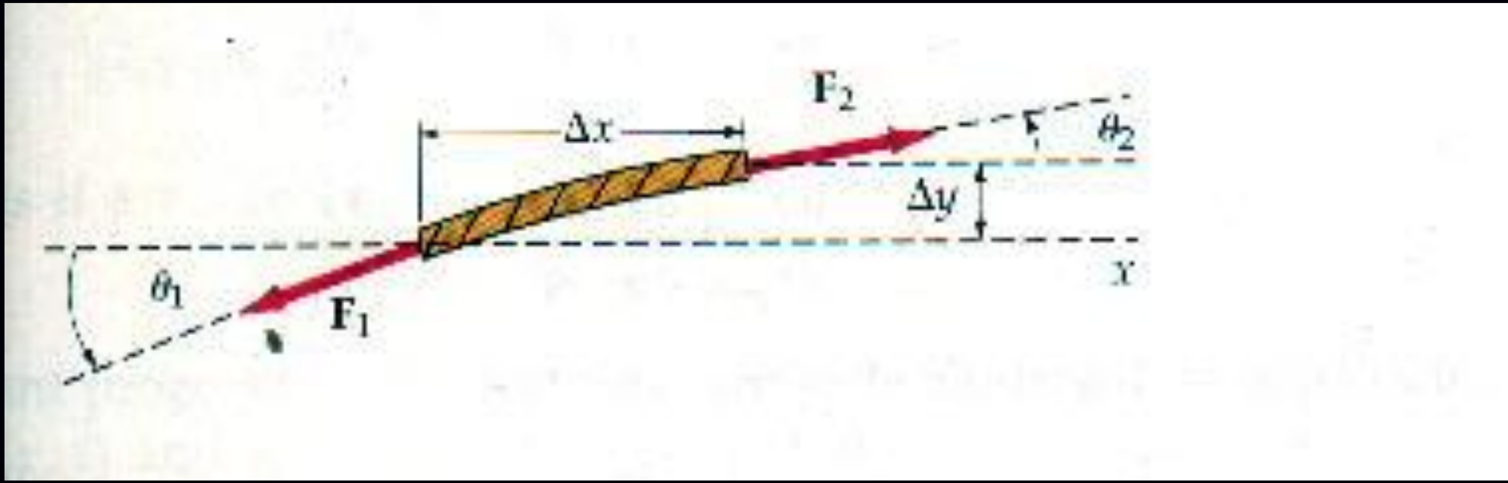
1. Energy of harmonic waves
2. Interference of mechanical waves
3. Standing waves

Energy of harmonic waves

Energy travels when waves travel

Average energy of medium

When transverse waves travel in some elastic medium:



Choose a segment infinite small

$$dV = Sdx \qquad dm = \rho Sdx$$

Its kinetic energy: $dW_k = \frac{1}{2} dm\, v^2$

$$\because \quad v = \frac{\partial y}{\partial t} = -A\omega \sin \omega (t - \frac{x}{u})$$

$$\therefore \quad dW_k = \frac{1}{2} (\rho\, dV) A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u})$$

Its potential energy:

$$dW_p = \frac{1}{2} G dV \left(\frac{dy}{dx} \right)^2$$

$$u = \sqrt{\frac{G}{\rho}} \qquad \rightarrow G = u^2 \rho$$

$$dW_p = \frac{1}{2} \rho u^2 \left(\frac{dy}{dx} \right)^2 \qquad \because \quad \frac{dy}{dx} = A \frac{\omega}{u} \sin \omega (t - \frac{x}{u})$$

$$\therefore \quad dW_p = \frac{1}{2} \rho \, dV A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u})$$

$$dW_k = dW_p$$

Total energy:

$$dW = dW_k + dW_p = \rho\, dV A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u})$$

the kinetic energy and potential energy of the infinitely small mass in the medium is the same.

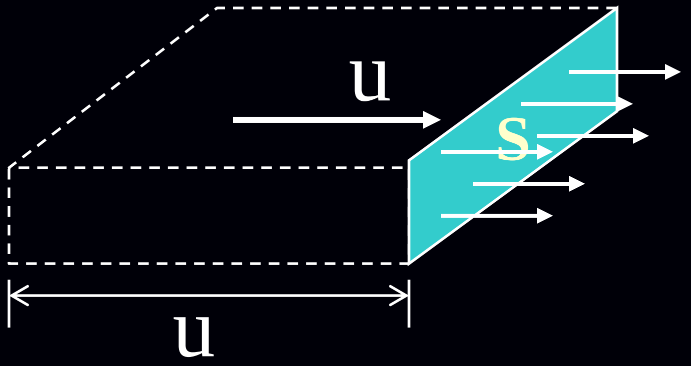
$$w = \frac{dW}{dV} = \rho A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u})$$

$$\overline{w} = \frac{1}{T} \int_0^T \rho A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u}) dt$$

Average energy density:

$$\boxed{\overline{w} = \frac{1}{2}\rho A^2\omega^2}$$

Energy flux: ***the energy pass certain cross section of medium in unit time.***



Average energy flux：

$$\overline{P} = \overline{w}uS$$

Energy flux density(Intensity)（波的强度）：

The average energy pass through a certain unit area along the direction that waves travel

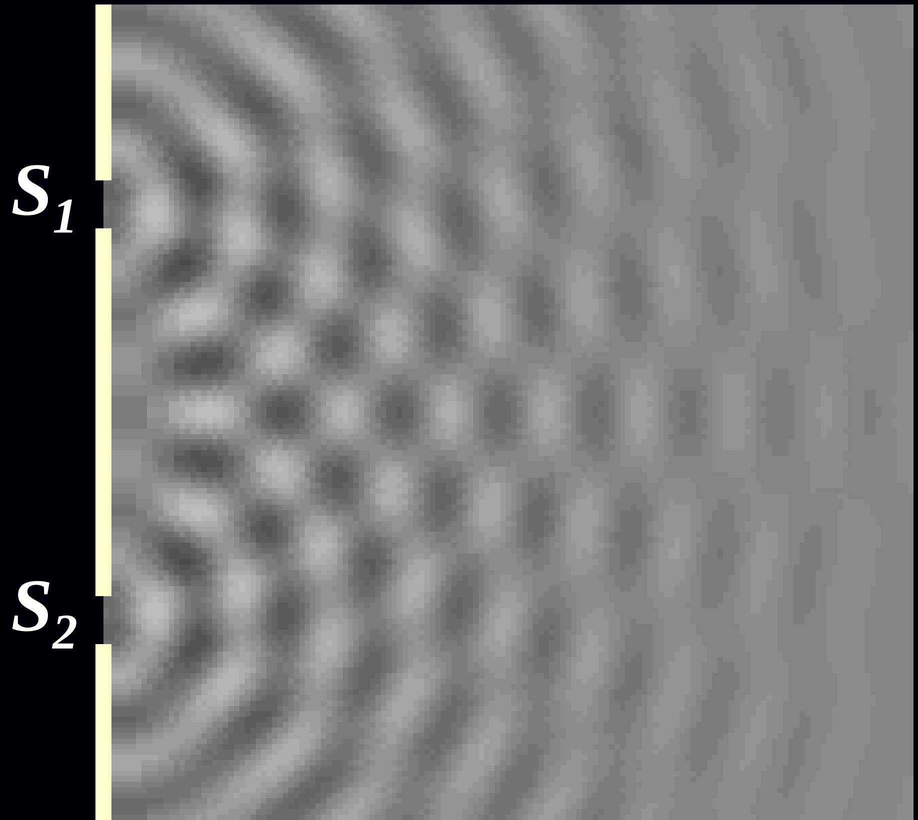
$$\boxed{I = \overline{w}u = \frac{1}{2}\rho u A^2 \omega^2}$$

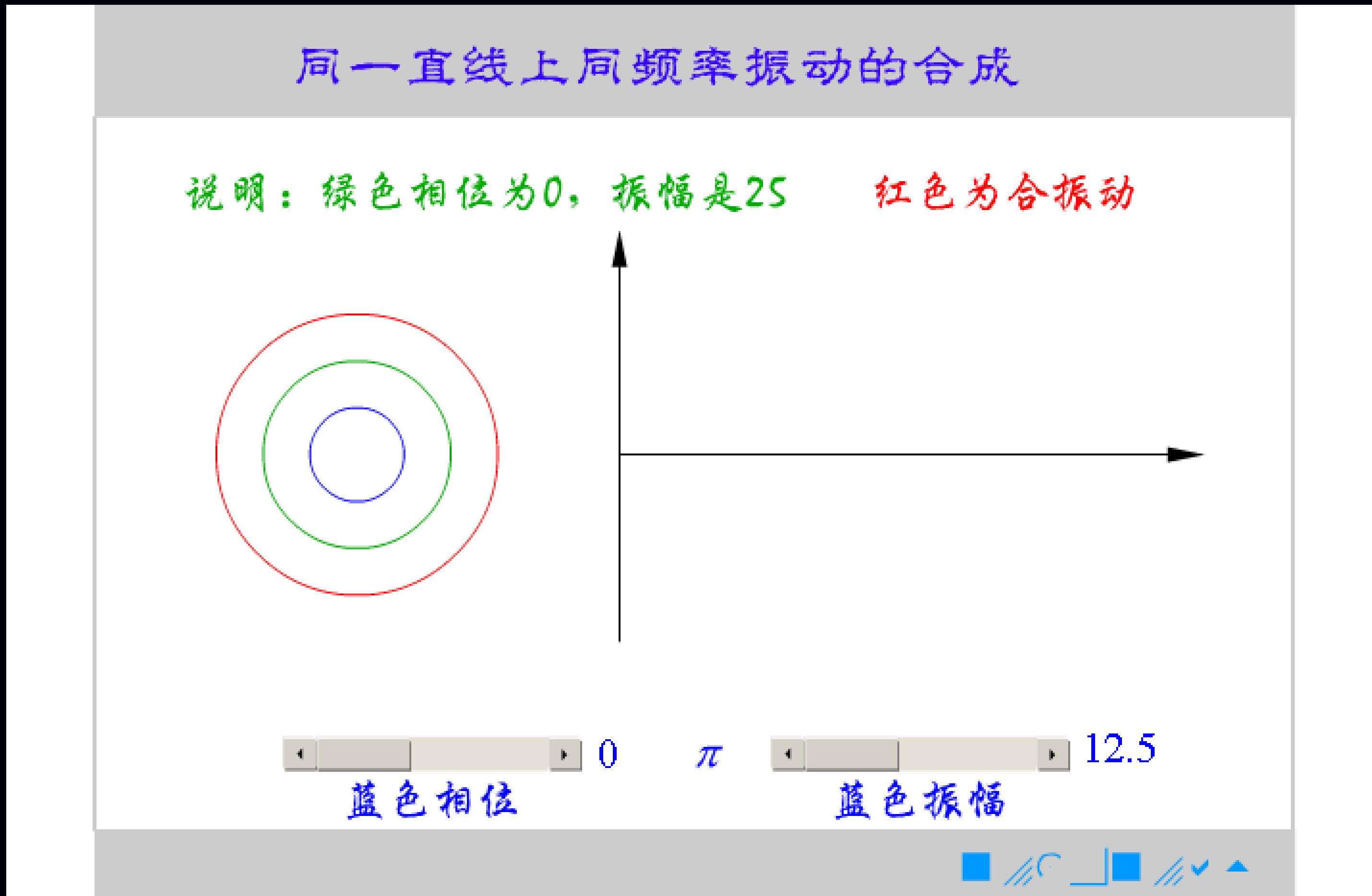
Interference of harmonic waves

Interference（干涉）：

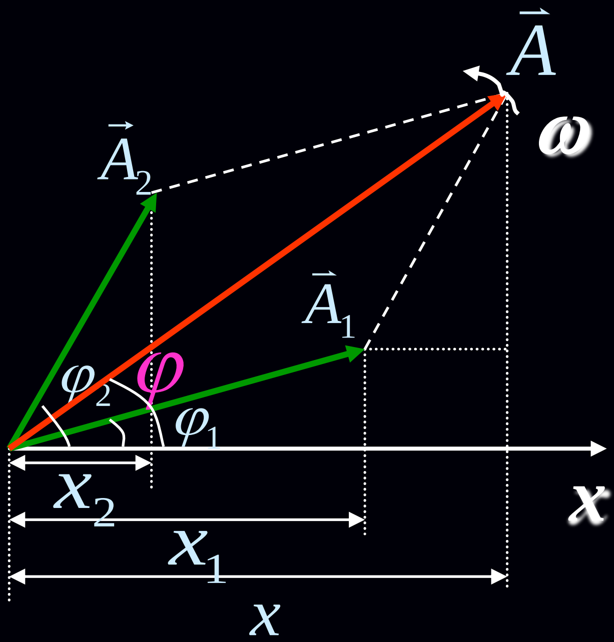
the various pattern caused by the superposition of harmonic waves.

Coherence（相干性）: the sources are coherence when they have same frequency and fixed phase difference





Superposition of oscillations with same frequency



$$\boxed{A=\sqrt{A_1^{\,2}+A_2^{\,2}+2A_1A_2\cos(\varphi_2-\varphi_1)}}$$

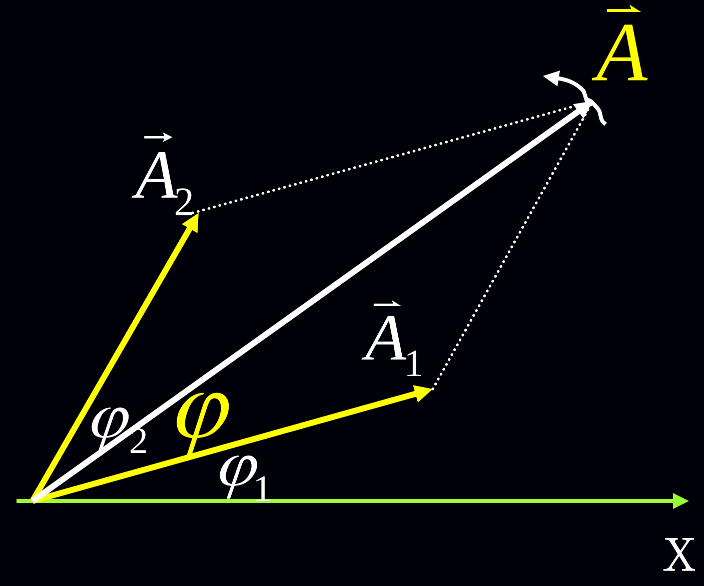
$$\boxed{\varphi=\tan^{-1}\frac{A_1\sin\varphi_1+A_2\sin\varphi_2}{A_1\cos\varphi_1+A_2\cos\varphi_2}}$$

Wave interference in one dimension

Two coherence waves , one is given as $y_1 = A_1 \cos(\omega t - kx + \varphi_1$

the other is given as $y_2 = A_2 \cos(\omega t - kx + \varphi_2)$

The resultant wave can be got by the help of phasor



When $A_1 = A_2$

$$y = 2A\cos\frac{\varphi_1 - \varphi_2}{2}\cos\left(\omega t + \frac{\varphi_1 + \varphi_2}{2}\right)$$

The interference is called constructive if $\varphi_1 = \varphi_2$

The interference is called destructive if $\varphi_1 - \varphi_2 = \pi$

In interference in two dimensions

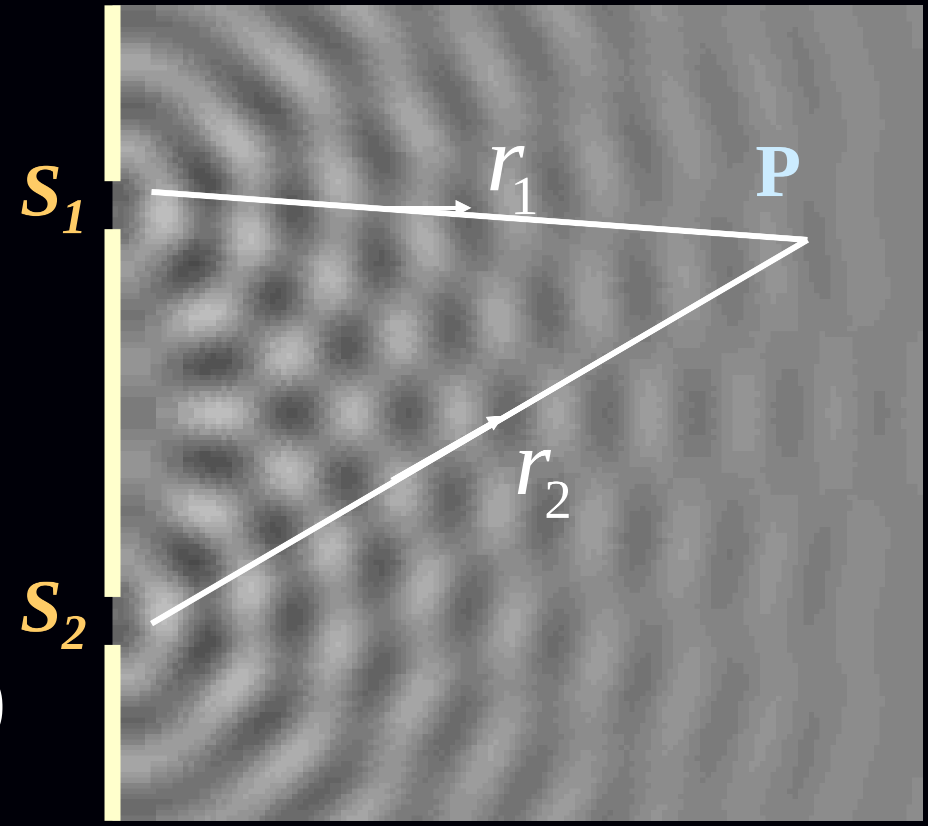
The equations for the sources:

$$S_1: \quad y_1 = A_{10}\cos(\omega t + \varphi_1)$$

$$S_2: \quad y_2 = A_{20}\cos(\omega t + \varphi_2)$$

$$y_1 = A_1\cos(\omega t + \varphi_1 - \frac{2\pi r_1}{\lambda})$$

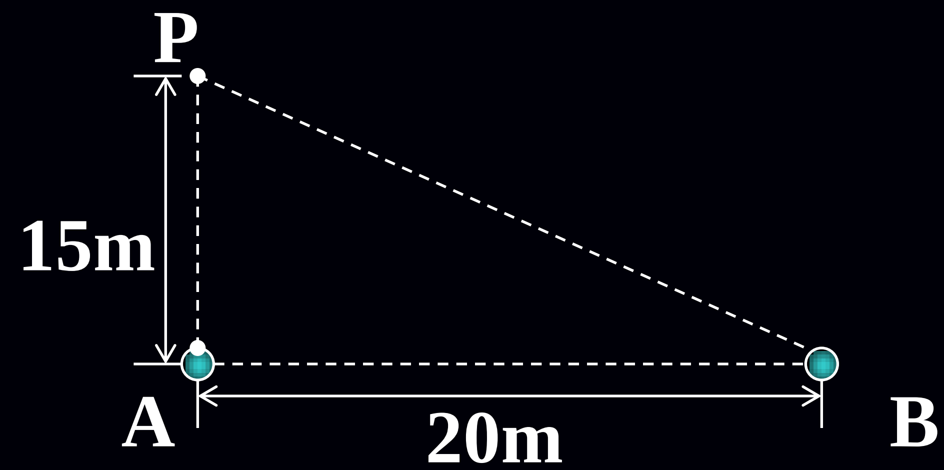
$$y_2 = A_2\cos(\omega t + \varphi_2 - \frac{2\pi r_2}{\lambda})$$



The phase difference:

$$\Delta\varphi = \varphi_2 - \varphi_1 - 2\pi(\frac{r_2 - r_1}{\lambda})$$

Example: A， B are two coherence sources, the amplitudes are 5cm， frequency is100Hz， wave speed is 10m/s。When A is in crest， B is in the trough， find the result of interference of these two waves at point P.



solution：

$$BP=\sqrt{20^2+15^2}=25m$$

$$\lambda=\frac{u}{\nu}=0.1m$$

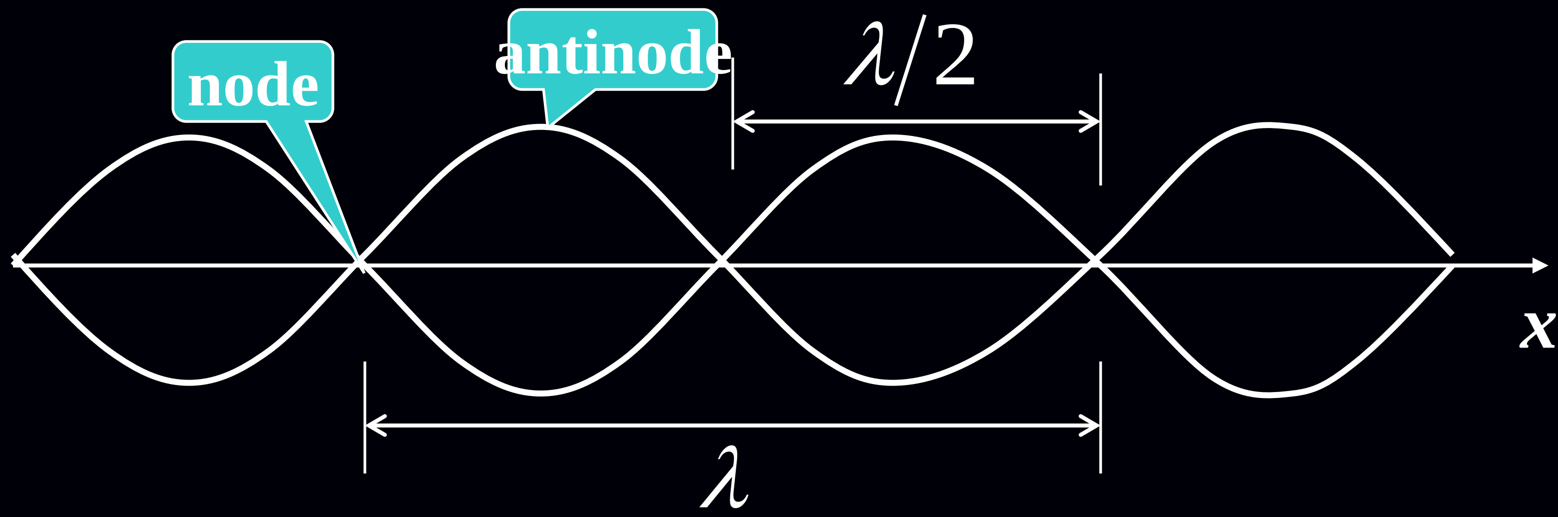
Assume the phase of A lead that of B π $\varphi_A-\varphi_B=\pi$

$$\Delta\varphi=\varphi'_B-\varphi'_A=-\pi-2\pi\frac{BP-AP}{\lambda}=-\pi-2\pi\frac{25-15}{0.1}$$

$$=-201\pi$$

In destructive interference

Standing waves（驻波）



The interference result by two coherent waves with amplitudes propagating in opposite direction.

$$y_1 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$y_2 = A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$y = y_1 + y_2 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

The equation for standing waves:

$$\boxed{y = 2A\cos\frac{2\pi}{\lambda}x\cos\frac{2\pi}{T}t}$$

The distance between two nearby node is $\lambda/2$

The particles between two nodes have the same phase angle.

The particles besides one node have phase difference of π .

The difference between traveling waves and standing waves

the amplitude does not change with position in traveling waves , while changes in standing waves

The energy travels in traveling waves , while does not in standing waves

Standing waves by reflection:

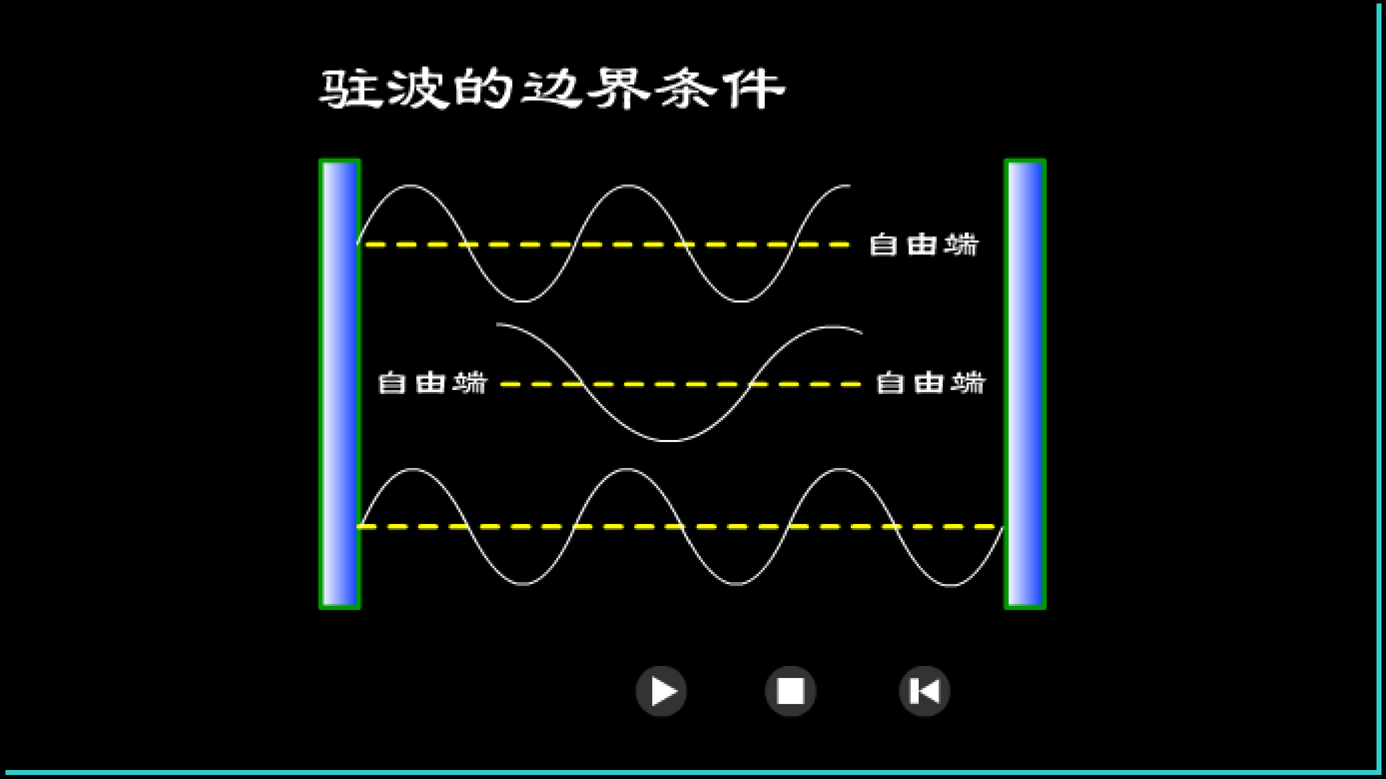
Half wavelength loss（半波损失）:

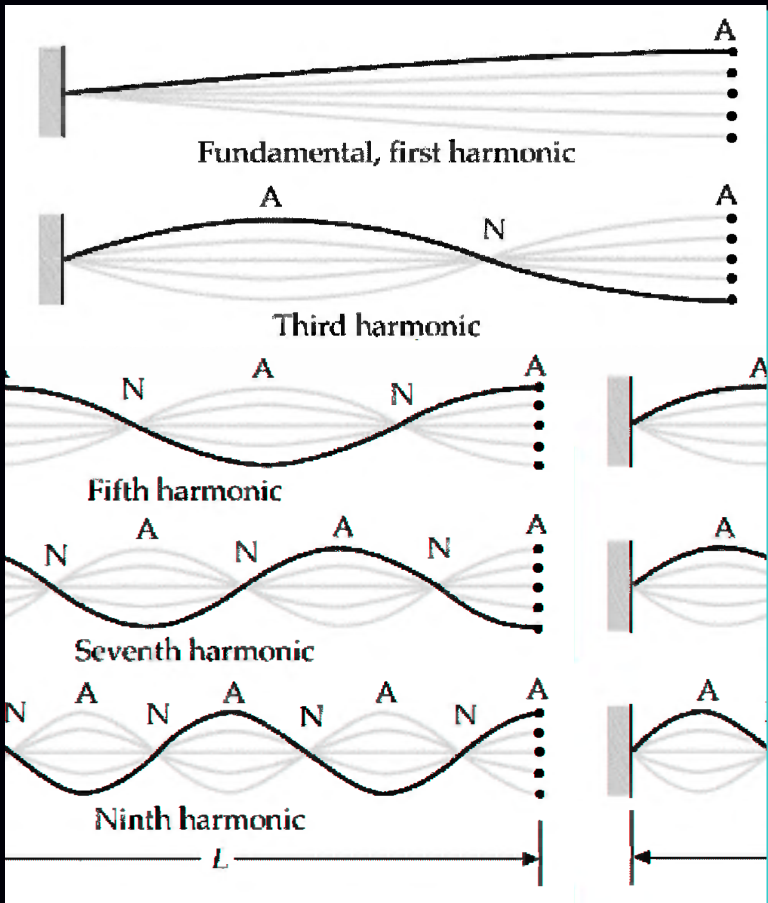
If a wave in medium 1 is incident on a boundary that separates medium 1 and medium 2, the product of density and speed of wave of medium 1 is less than that of medium 2, the reflected wave is inverted. The particle on the boundary is node.

if the product of density and speed of wave of medium 1 is larger than that of medium 2, the reflected wave is not inverted. The particle on the boundary is antinode.

Standing waves in rope

One end is free, the another is not:





The condition for standing waves

$$l = n\frac{\lambda_n}{4} \qquad n = 1,3,5\cdots$$

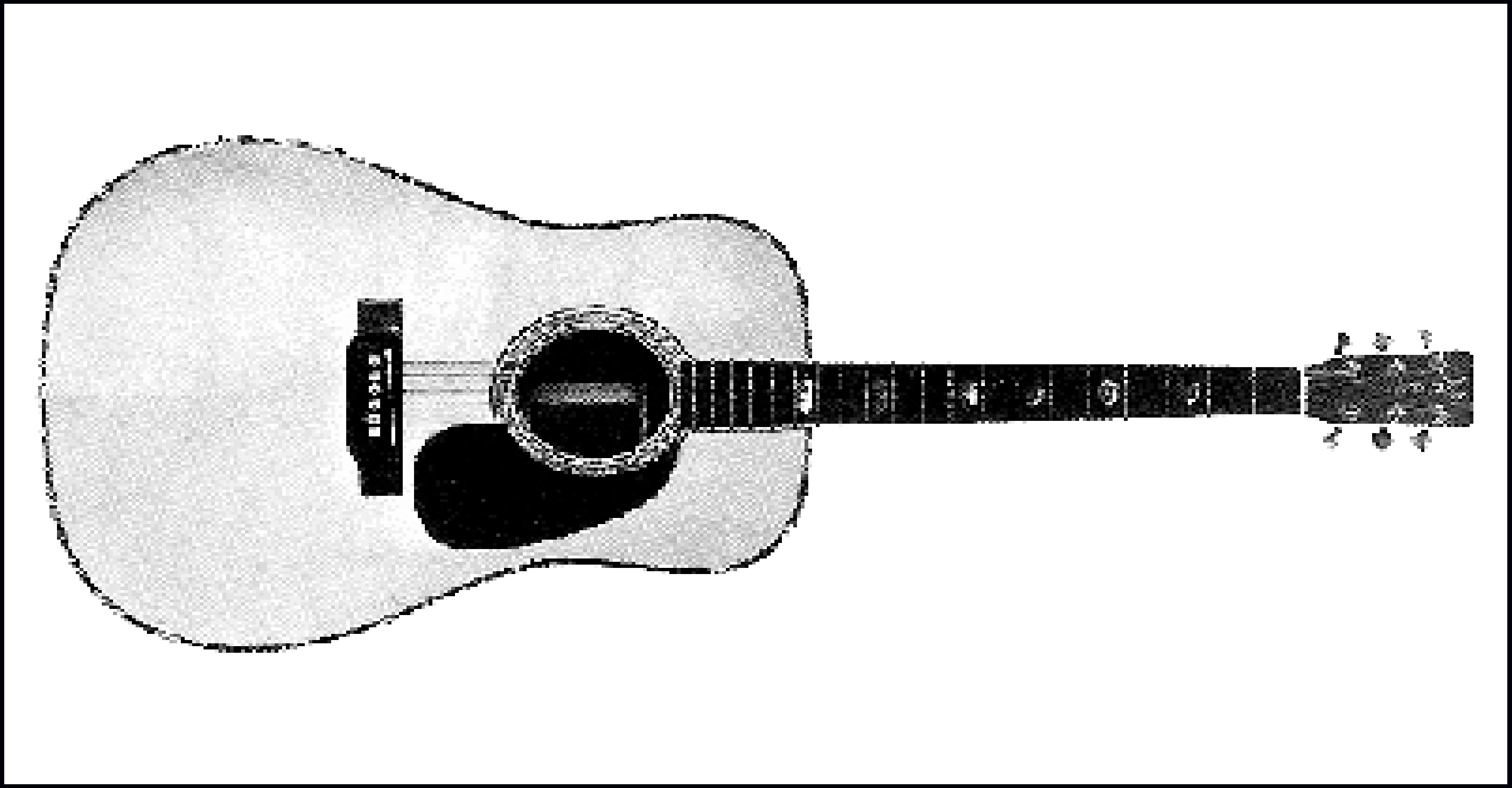
Bell Labs Wave Machine

STANDING WAVES

MIT Department of Physics
Technical Services Group

How did the player make different sound from the trombone?

Both ends are fixed



The condition for standing waves

$$l = n\frac{\lambda_n}{2}$$

Example: a harmonic wave travels along a string, the equation that describes the motion of it is given as:

$$y_1 = 2.0\times 10^{-2}\cos[2\pi(\frac{t}{0.02}-\frac{x}{20})+\frac{\pi}{3}]\ (SI)$$

Another wave is traveling in opposite direction, the resultant standing waves has node at x=0, find the equation that describes the motion of this wave.

solution:

$$y_2 = 2.0\times10^{-2}\cos[2\pi(\frac{t}{0.02}+\frac{x}{20})+\varphi]$$

$$y = y_1 + y_2 = 4.0\times10^{-2}\cos[\frac{1}{2}(\frac{2x}{20}+\varphi-\frac{\pi}{3})]\cos[\frac{1}{2}(\frac{4\pi t}{0.02}+\varphi+\frac{\pi}{3})]$$

$$\frac{1}{2}(\varphi-\frac{\pi}{3})=\frac{\pi}{2} \qquad \to \varphi = \frac{4\pi}{3}$$

$$y_2 = 2.0\times10^{-2}\cos[2\pi(\frac{t}{0.02}+\frac{x}{20})+\frac{4\pi}{3}]$$