# The Kinetic Theory of Gas

# Agenda Today

- Ideal gas
- Pressure and temperature ( micro interpretation)
- Internal energy of idea gas
- Distribution law  of  molecular speed  for ideal gas
-  mean free path

The ideal-gas law（理想气体状态方程）

$$PV = \frac{m}{M}RT$$

M: molar mass（摩尔质量）

R: universal gas constant（普适气体常数）
R=8.31J/mol˙k

Avogadro's number:

$$N_A = 6.022 \times 10^{23}\ mol^{-1}$$

The number of moles

$$\frac{m}{M} = \frac{N}{N_A}$$

Other forms of idea-gas Law

$$\boxed{PV = NkT}$$

K : Boltzmann's constant $1.38\times10^{-23} J/K$

$$\rho = \frac{M}{RT}P$$

The molecular interpretation of temperature:

the kinetic theory of gases

1 the gas consists of a large number of molecules that make elastic collisions with each other and with the walls of the container

2 the molecules are separated, on the average, by the distances that are very large compared with their diameters

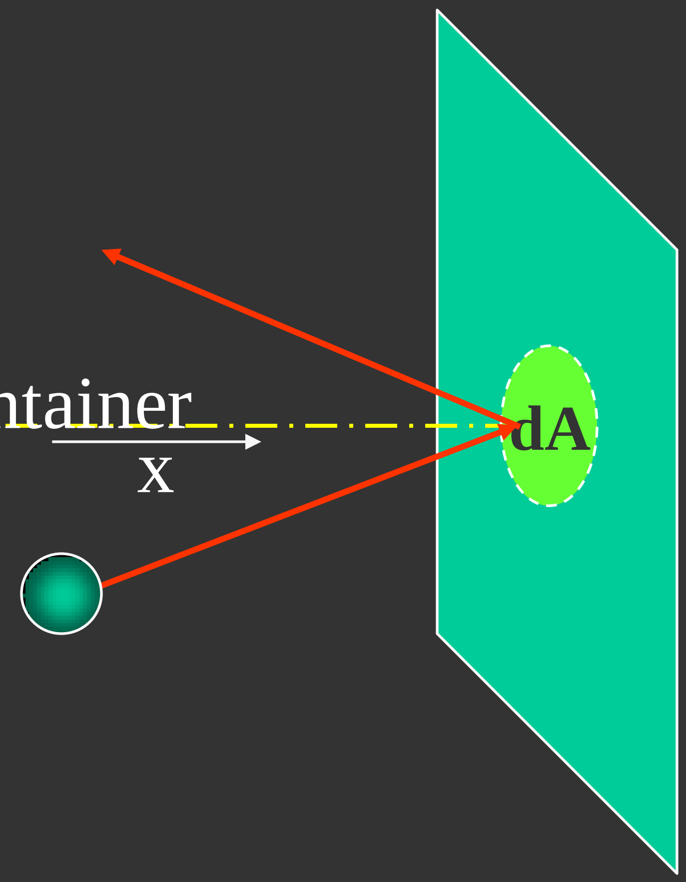
3 the molecules did not exert force on each other except when they collide

4 there is no preferred position for a molecule and no preferred direction for velocity, when there is no external force

Suppose the gas is held in a cube container, one molecule moves ceaselessly

The increase of molecule's momentum

$$-\mu v_{ix} - \mu v_{ix} = -2\mu v_{ix}$$




The impulse on the wall of the container

$$2\mu v_{ix}$$

**The number of collision the molecule has with the wall in dt**

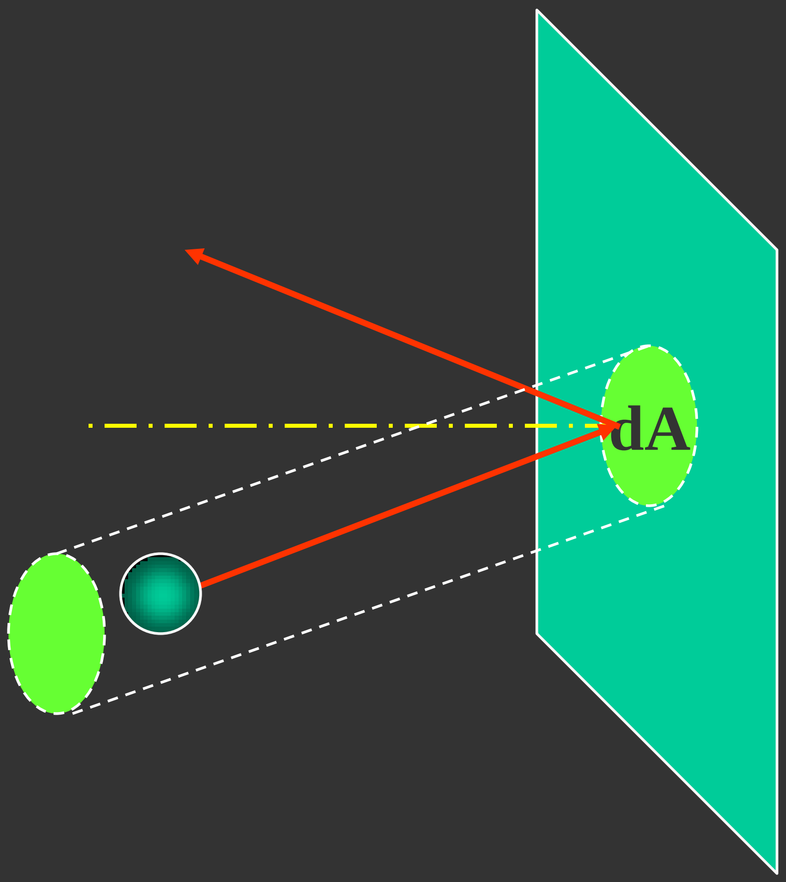
$$\frac{1}{2l_x} v_{ix} dt$$

The impulse of all the molecule:

$$dI = \sum \frac{\mu v_{ix}^2}{l_x} dt$$

**force** $$dF = \frac{dI}{dt}$$

$$= \sum \frac{\mu v_{ix}^2}{l_x}$$



**pressure:**

$$P = \frac{dF}{A} = \sum \frac{\mu v_{ix}^2}{A l_x} = \mu \frac{N \overline{v_{ix}^2}}{V}$$

**since**

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3} \overline{v^2}$$

$$P = \mu n \overline{v_x^2} = \frac{1}{3} \mu n \overline{v^2}$$

$$\because \quad \bar{\varepsilon} = \frac{1}{2}\mu\overline{v^2}$$

$$\boxed{P = \frac{2}{3}n\bar{\varepsilon}}$$

$$\because \quad P = nkT = \frac{2}{3}n\bar{\varepsilon}$$

$$\boxed{\bar{\varepsilon} = \frac{3}{2}kT}$$

$$\because \quad \bar{\varepsilon} = \frac{1}{2}\mu\overline{v^2} = \frac{3}{2}kT$$

**Root mean square speed（方均根速率）**

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{\mu}}$$

$$\because \quad \frac{k}{\mu} = \frac{kN_A}{\mu N_A} = \frac{R}{M}$$

Rms speed:

$$\boxed{\sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{\mu}} = \sqrt{\frac{3RT}{M}}}$$

Degree of freedom 自由度

For monatomic molecule $i=3$

diatomic molecule $i=5$

molecule with atom more the three $i=6$

# The equipartition theorem ( 能量按自由度均分原理）

The contribution of each degree of freedom to the average energy of molecule is kT/2

The average energy for a molecule

$$\overline{\varepsilon}_k = \frac{i}{2}kT$$

For monatomic molecule i=3

diatomic molecule i=5

molecule with atom more the three i=6

The internal energy of ideal-gas:理想气体内能

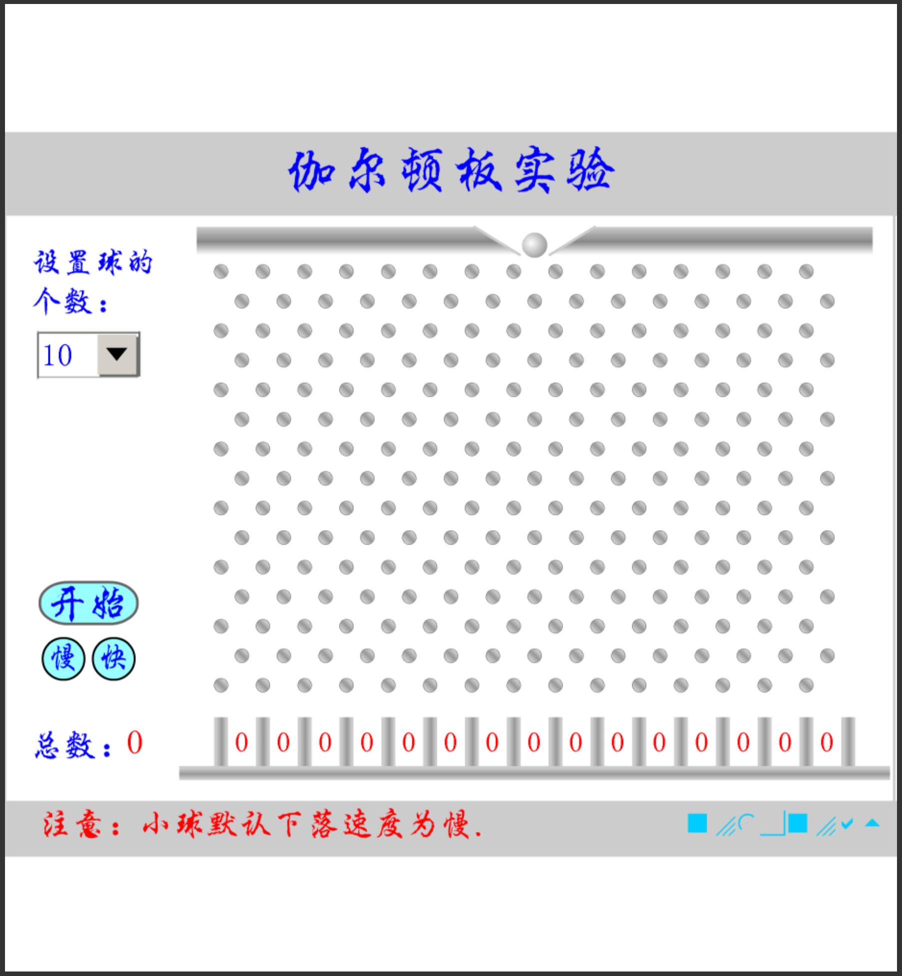
The kinetic energy of all the molecules in the gas

$$\boxed{E = \frac{m}{M} E_{mol} = \frac{m}{M} \frac{i}{2} RT}$$

Where m is the mass of the gas

M is the molar mass of the gas

# Probability distribution（概率分布）




Discrete distribution

$$P_i = \frac{N_i}{N}$$

Normalization condition（归一化条件）

$$\sum_i P_i = 1$$

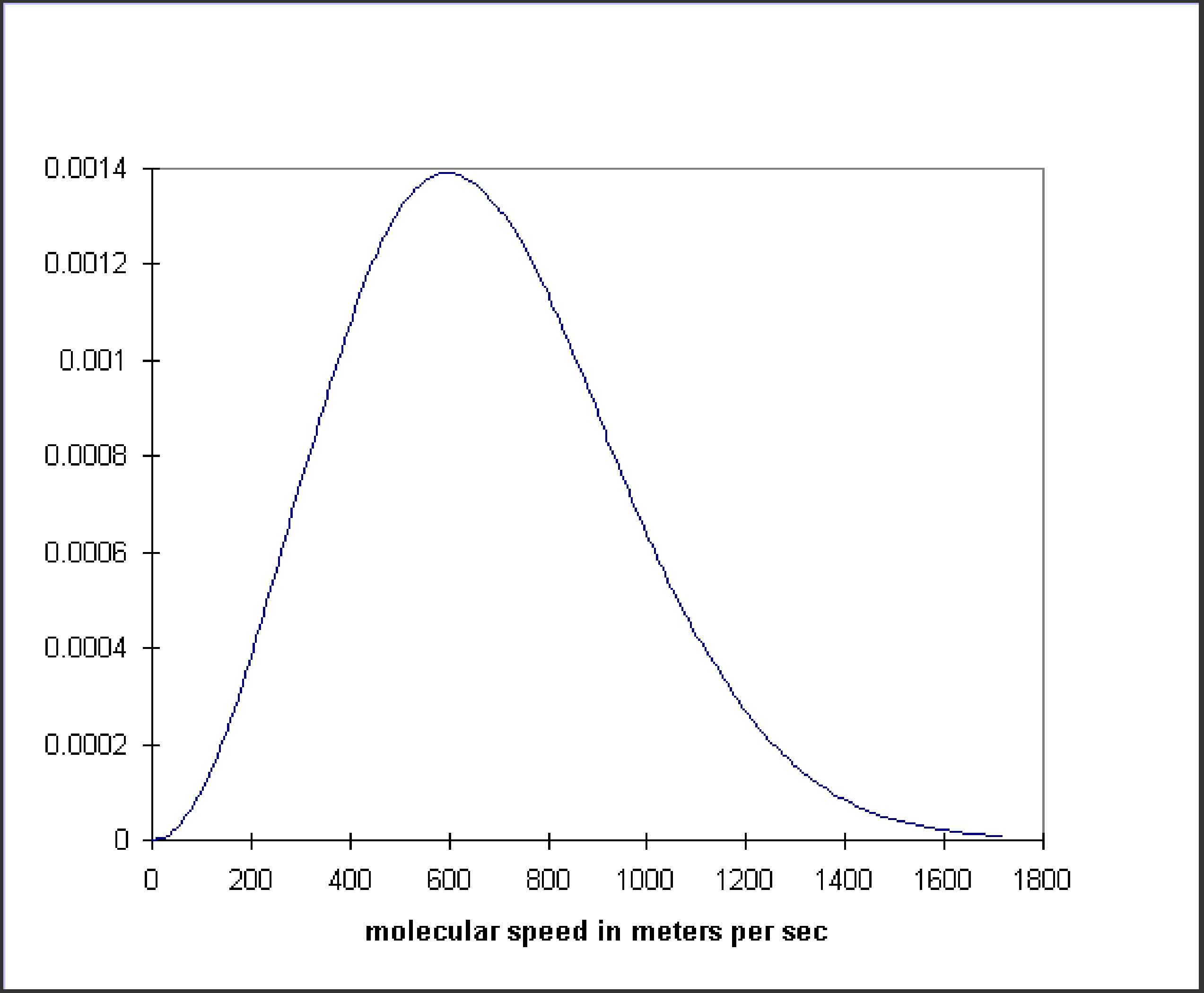
The average value

$$\bar{x} = \frac{1}{N}\sum_i x_i N_i = \sum_i x_i P_i$$

The average of square

$$\overline{x^2} = \frac{1}{N}\sum_i x_i^2 N_i = \sum_i x_i^2 P_i$$

# Continuous distribution



Probability distribution function

$$f(v)=\lim_{\Delta v\to 0}\frac{\Delta N}{N\Delta v}=\frac{1}{N}\frac{dN}{dv}$$

$$\boxed{f(v)dv=\frac{dN}{N}}$$

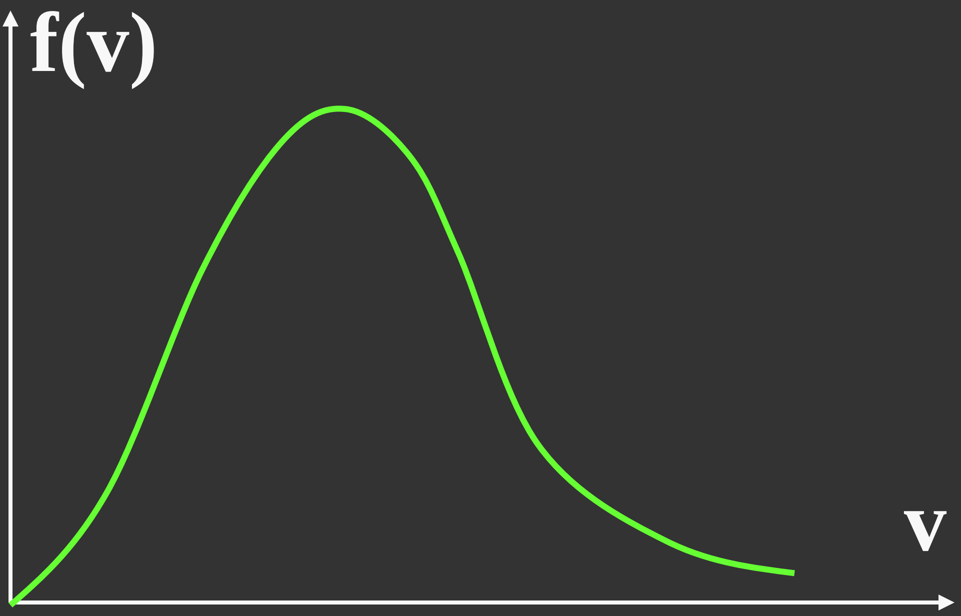
F(v)dv stands for the probability of a molecule's speed  is between v to v+dv

# The Maxwell distribution（麦克斯韦速率分布函数）

$$f(v) = 4\pi\left(\frac{\mu}{2\pi kT}\right)^{3/2} e^{-\frac{\mu v^2}{2kT}} v^2$$

$$f(v)dv = \frac{dN}{N}$$

$$\int_0^\infty f(v)dv = 1$$



The average speed:（算术平均速率）

$$\boxed{\bar{v} = \frac{\int v dN}{N} = \int v f(v) dv}$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi\mu}} = \sqrt{\frac{8RT}{\pi M}} \approx 1.60\sqrt{\frac{RT}{M}}$$

The most likely speed（最可几速率）：

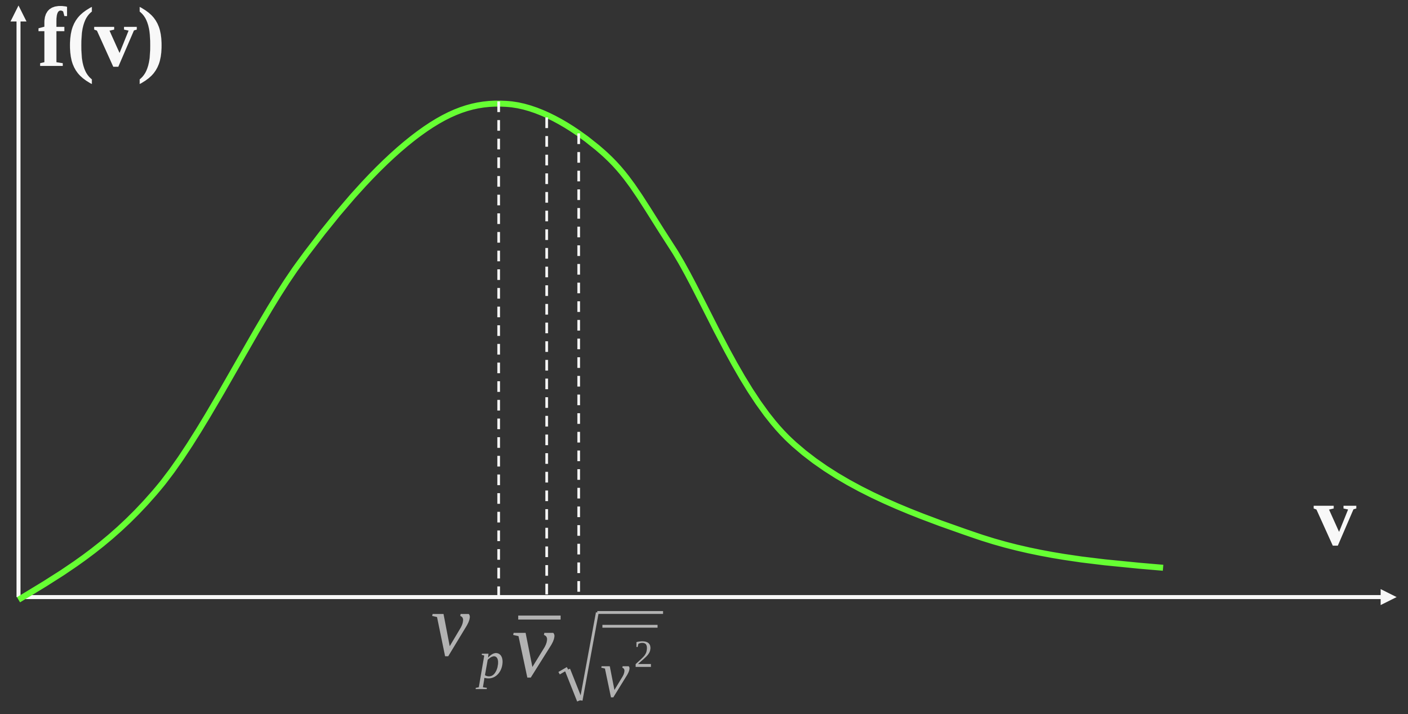
$$\frac{d}{dv} f(v) = 0$$

$$v_p = \sqrt{\frac{2kT}{\mu}} = \sqrt{\frac{2RT}{M}} \approx 1.41\sqrt{\frac{RT}{M}}$$

The root mean square speed（方均根速率）

$$\overline{v^2} = \int v^2 f(v) dv$$

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{\mu}} = \sqrt{\frac{3RT}{M}} \approx 1.73\sqrt{\frac{RT}{M}}$$



**With the conditions given， find the number of Nitrogen molecule whose speed is from 500 to 501ms$^{-1}$(V=1m$^3$)**

$$T = 273.15K \qquad P = 1.013\times10^5\ Pa$$

$$M = 28\times10^{-3}\ kg\cdot mol^{-1}$$

**solution：**

$$\mu = \frac{M}{N_A} = \frac{28\times10^{-3}}{6.022\times10^{23}} = 4.65\times10^{-26}kg$$

$$n = \frac{P}{kT} = \frac{1.013\times10^5}{1.38\times10^{-23}\times273.15} = 2.7\times10^{25}\ m^{-3}$$

$$\frac{\Delta n}{n} = \frac{\Delta N}{N} = \left(\frac{\mu}{2\pi kT}\right)^{3/2} e^{-\frac{\mu v^2}{2kT}} \cdot 4\pi v^2 \Delta v$$

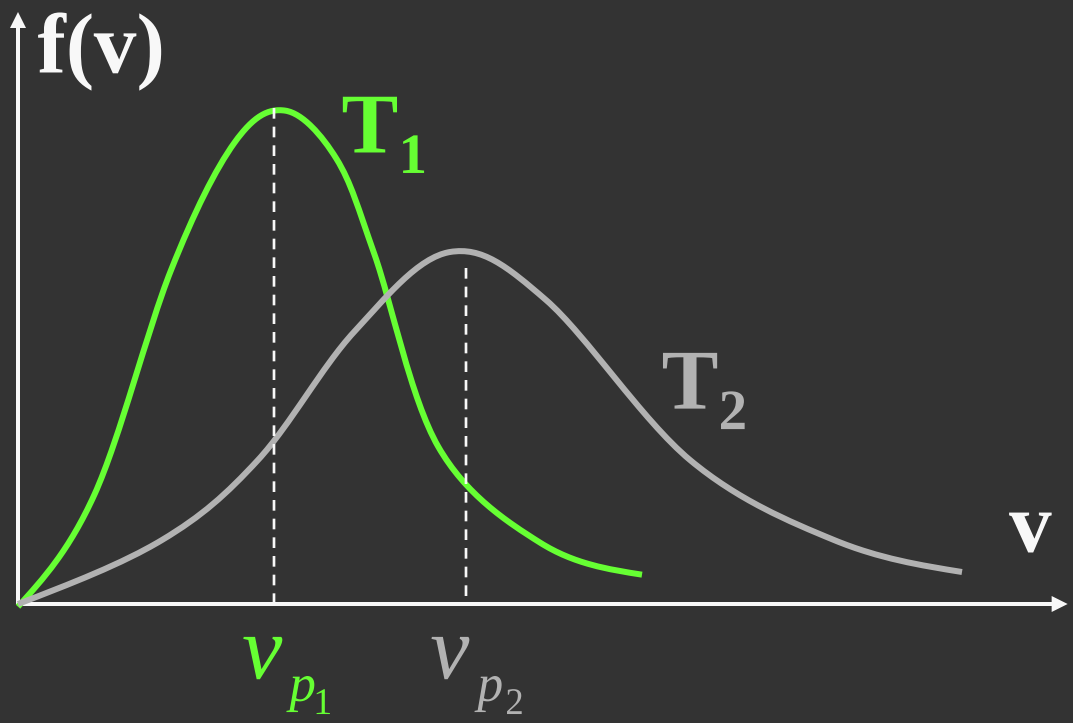
$$v = 500\, m \cdot s^{-1}, \Delta v = 1\, m \cdot s^{-1}$$

$$\frac{\Delta n}{n} = 1.85 \times 10^{-3}$$

$$\Delta n = 1.85 \times 10^{-3} \times 2.7 \times 10^{25} = 5.0 \times 10^{22}\ m^{-3}$$

**The maxwell distribution function of oxygen and hydrogen are given below, which one is the function for hydrogen?**

$$v_p = \sqrt{\frac{2RT}{M}}$$



Maxwell-Bolzmann distribution

$$f(v) = \frac{1}{Z} e^{-\frac{E_k + E_p}{kT}}$$

$$\boxed{n = n_o e^{-\frac{\mu gz}{kT}}}$$

$$\because \quad p = nkT\,,\ p_o = n_o kT$$

$$\boxed{p = p_o e^{-\frac{\mu gz}{kT}} = p_o e^{-\frac{M gz}{RT}}}$$

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$$p = p_o e^{-\frac{\mu g z}{kT}} = p_o e^{-\frac{M g z}{RT}}$$

Why oxygen bottles is needed in maintain climbing?

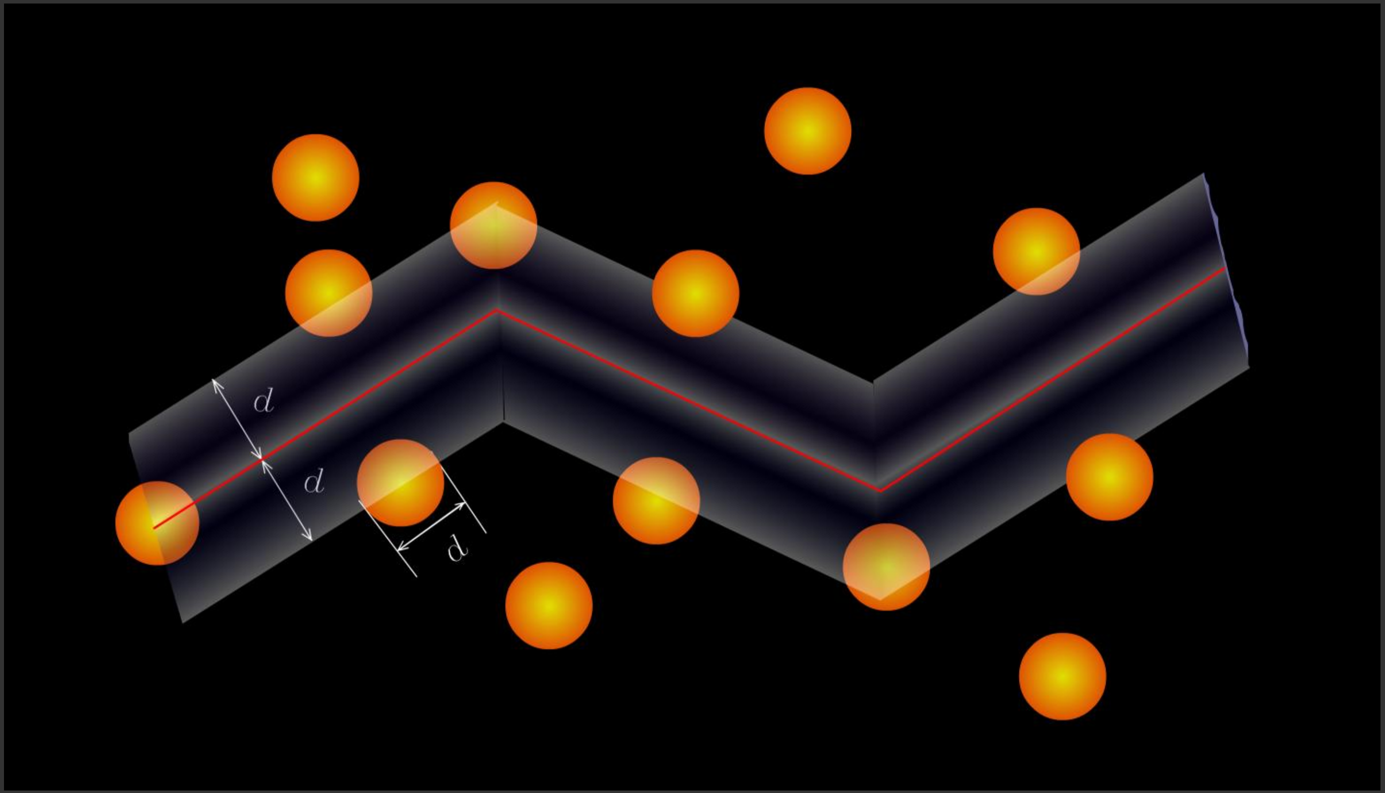
Transportation phenomena输运现象

Mean free path（平均自由程）

The average distance a molecule may travel before the next collision

collision frequency(平均碰撞频率）

The number of collision in unit time




Diameter of molecule： d， density of molecule： n

**Collision frequency:** $\bar{z} = \sqrt{2}\pi d^2 n\bar{v}$

**Mean free path:**

$$\bar{\lambda} = \frac{\bar{v}}{\bar{z}} = \frac{1}{\sqrt{2}\pi d^2 n}$$

$\because \quad p = nkT$

$$\bar{\lambda} = \frac{kT}{\sqrt{2}\pi d^2 p}$$

**mean free path is proportional to the density of the gas inversely**

**Find the collision frequency of hydrogen gas at standard state。（the diameter of molecule d = 2×10$^{-10}$m)**

**solution：**

$$\bar{v}=\sqrt{\frac{8RT}{M\pi}}=\sqrt{\frac{8\times 8.31\times 273}{2\times 10^{-3}\pi}}=1.70\times 10^{3}\,m\cdot s^{-1}$$

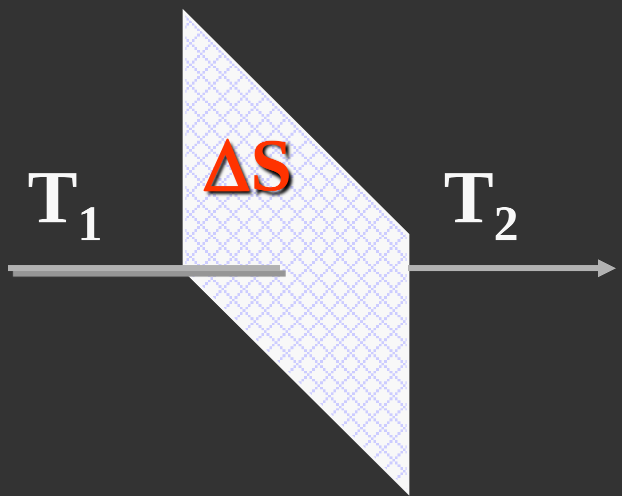
$$n=\frac{P}{kT}=\frac{1.013\times 10^{5}}{1.38\times 10^{-23}\times 273}=2.69\times 10^{25}\text{个}m^{-3}$$

$$\bar{\lambda}=\frac{1}{\sqrt{2}\pi d^{2}n}=2.14\times 10^{-7}\,m$$

$$\bar{z}=\frac{\bar{v}}{\bar{\lambda}}=7.95\times 10^{9}\,s^{-1}$$

## Heat conduction

The law of heat conduction（热传导方程）




$$\frac{\Delta Q}{\Delta t} = -K\frac{dT}{dx}\Delta S$$

**Thermal conductivity(热导率）**

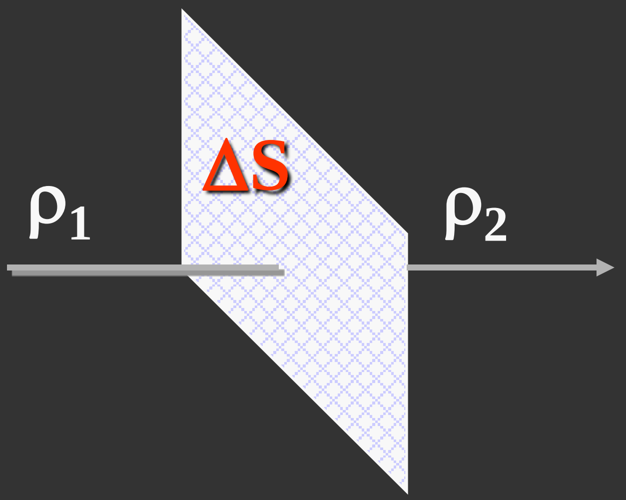
$$k = \frac{1}{3}\rho\bar{v}\bar{\lambda}C_v$$

# The diffusion of mass

Fick’s Law

（菲克定律）

$$\frac{\Delta m}{\Delta t} = -D\frac{d\rho}{dx}\Delta S$$




**Diffusion constant**

扩散系数

$$D = \frac{1}{3}\bar{v}\bar{\lambda}$$