# Electric potential

# Agenda today

electric potential

equipotentials

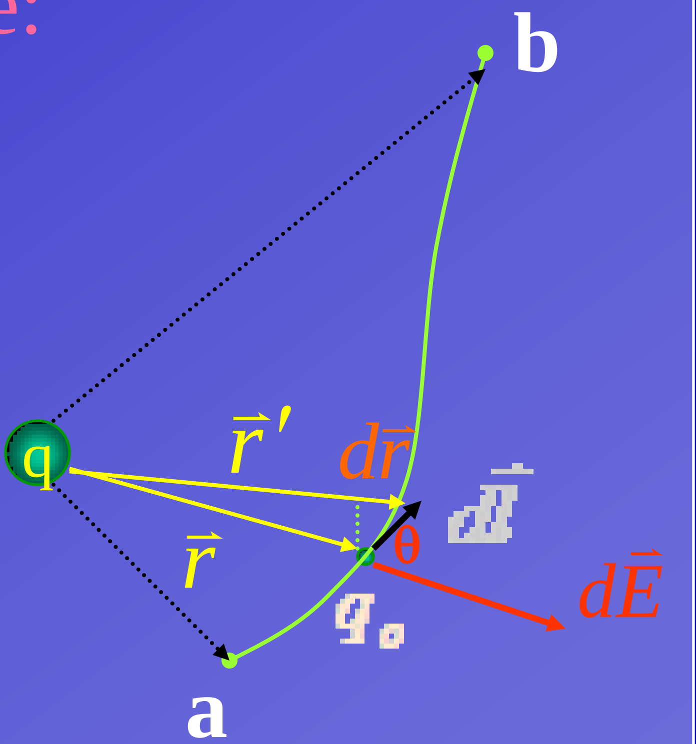
relationship between electric potential and field strength

electric potential and electric field around conductor

# Work done by electric force:

$$dA = q_o \vec{E} \cdot d\vec{l} = q_o E \cos\theta \, dl$$

$$E = \frac{q}{4\pi\varepsilon_o r^2}$$




$$dA = \frac{q_o q}{4\pi\varepsilon_o r^2} \cos\theta \, dl = \frac{q_o q}{4\pi\varepsilon_o r^2} dr$$

$$A_{ab} = \int_{r_a}^{r_b} \frac{q_o q}{4\pi\varepsilon_o r^2} dr = \frac{q_o q}{4\pi\varepsilon_o} \left( \frac{1}{r_a} - \frac{1}{r_b} \right)$$

The work done by electric force is independent of the path taken by the charge, thus static electric force is conservative force.

$$A = \oint_l q_o \vec{E} \cdot d\vec{l} = 0$$

$$\boxed{\oint_l \vec{E} \cdot d\vec{l} = 0}$$

# Electric potential

Electric potential energy

Point charge q moves in static electric field with no other force exerted on the charge except the static electric force from a to b:

$$\Delta E_p = -\Delta E_k = -\int_a^b q\,\vec{E}\cdot d\vec{l}$$

Electric potential difference:

$$\Delta U_{ba} = \frac{\Delta E_p}{q} = -\int_a^b E\,d$$

If point a is chosen as the reference point:

$$U_b = \Delta U = -\int_{ref}^{b} E\,dl = \int_b^{ref} E\,d$$

Thus, electric potential difference can expressed as $U_{ba}=U_b-U_a$

In most cases, U is set as zero , when r approaches infinite.

# The unit of electric potential

Volt(V) 1V=1J/C

1V/M=1N/C

Electric potential distribution for field of one single charge

$$U_a = \int_a^{\infty} \vec{E} \cdot d\vec{l} = \int_{r_a}^{r_{\infty}} \frac{q}{4\pi\varepsilon_o r^2} dr = \frac{q}{4\pi\varepsilon_o r_a}$$

$$U_a = \frac{q}{4\pi\varepsilon_o r_a}$$

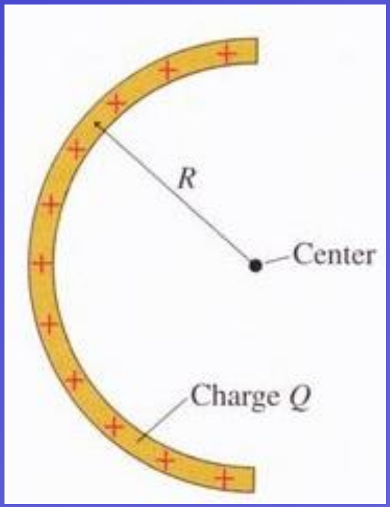
# Potential in a system of charges

1 If the electric field is known, the definition equation should be used

2 If the electric field is unknown, then use the superposition principle.

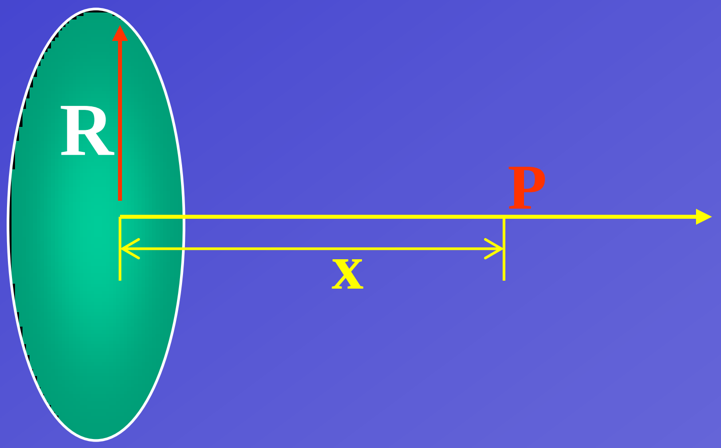
$$U = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

. || **FIGURE P29.71** shows a thin rod with charge $Q$ that has been bent into a semicircle of radius $R$. Find an expression for the electric potential at the center.



Example 2. The charged disk. Find the electric potential at any given point along the axis of the disk uniformly charged with charge Q.

We divide the disk into a series of concentric rings with radius r and width dr




For each ring , the charge it has:

$$dq = \sigma dA = \sigma 2\pi r\, dr$$

By integrating over all the rings in the disk

$$dU = \frac{dq}{4\pi\epsilon_0\sqrt{r^2+x^2}} = \frac{2\pi\sigma r\,dr}{4\pi\epsilon_0\sqrt{r^2+x^2}}$$

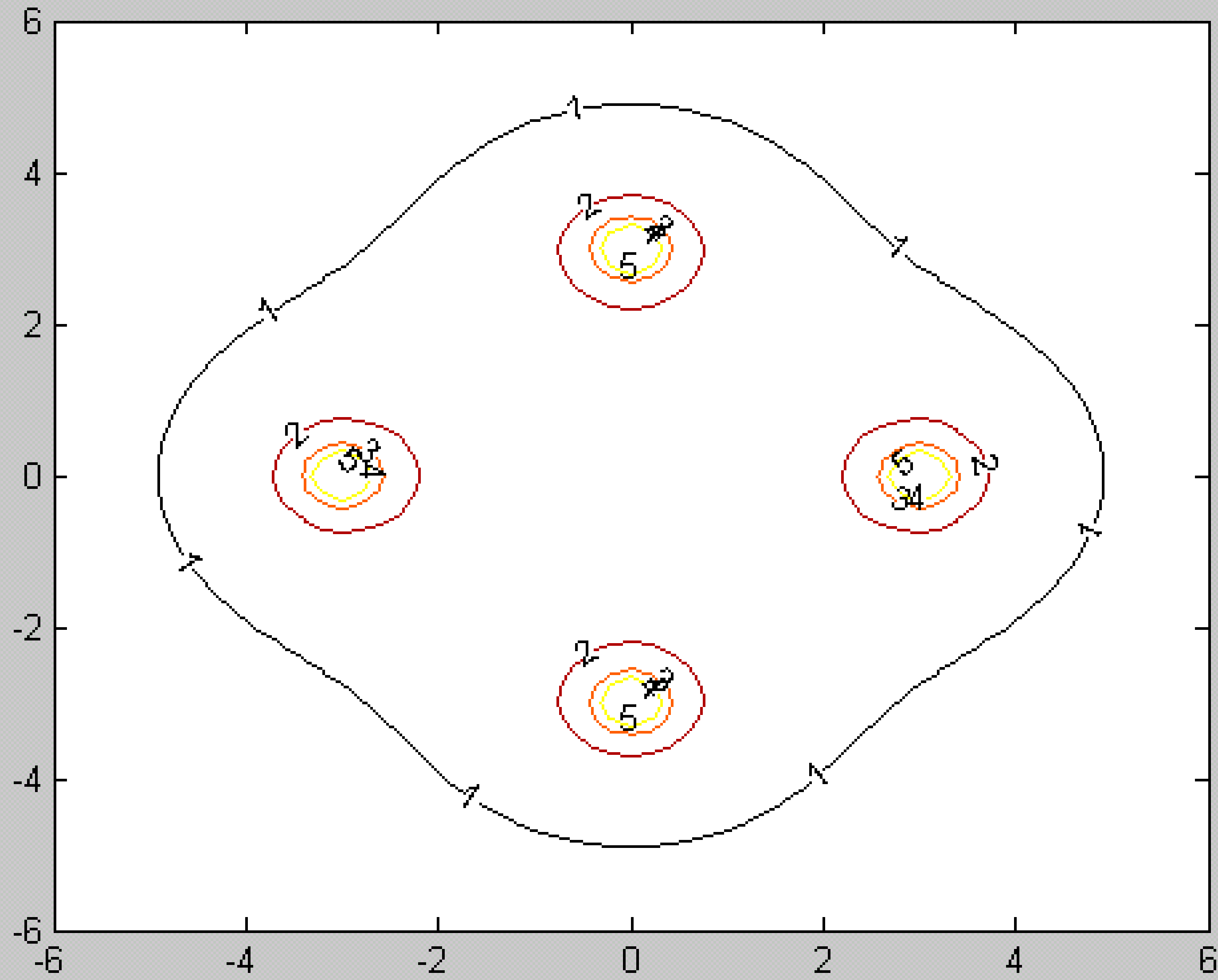
$$U = \int_0^R \frac{\sigma r\,dr}{2\epsilon_0\sqrt{r^2+x^2}} = \frac{\sigma}{2\epsilon_0}\sqrt{r^2+x^2}\,\Big|_0^R = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+x^2}-x\right)$$

Where $\sigma = Q/\pi R^2$

# Equipotential surface (等勢面)

surfaces where the electric potential of a charge distribution has constant value

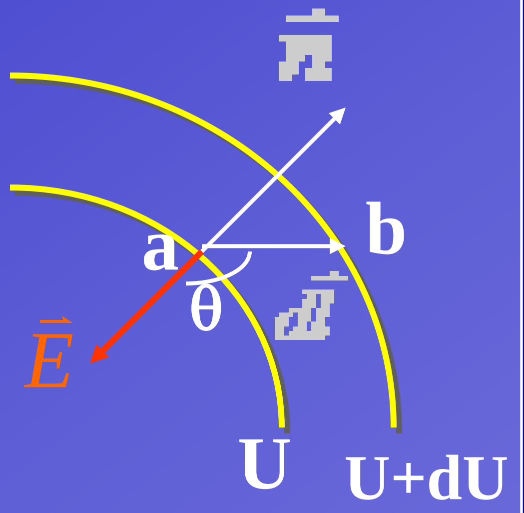
The electric field must be everywhere perpendicular to the equipotential surface



6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

# Find the field from potential

The electric field points along the shortest direction from one equipotential surface to the next one.




$$dA_{ab} = q_o\left[U - \left(U + dU\right)\right] = q_o\vec{E}\cdot d\vec{l}$$

$$-dU = E\cos\theta\, dl$$

$$E\cos\theta = E_l = -\frac{dU}{dl}$$

$$E_x = -\frac{\partial U}{\partial x} \quad E_y = -\frac{\partial U}{\partial y} \quad E_z = -\frac{\partial U}{\partial z}$$

Gradient operator(梯度算子)

$$E = -gradU = -\nabla U$$

$$\nabla \equiv i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k} = -(\frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y}\vec{j} + \frac{\partial U}{\partial z}\vec{k})$$

Example. A potential distribution in space is given as $U=Axy^2-Byz$, where A and B are constant. Find the electric field.

Solution:

Find the partial derivatives first

$$\frac{\partial U}{\partial x} = Ay^2$$

$$\frac{\partial U}{\partial y} = 2Axy - Bz$$

$$\frac{\partial U}{\partial z} = -By$$

The electric field is therefore:

$$E = -Ay^2\mathbf{i} - (2Axy - Bz)\mathbf{j} + By\mathbf{k}$$

# Potential and electric field around conductor

Conductor in static equilibrium state:

1 has no current flowing inside it

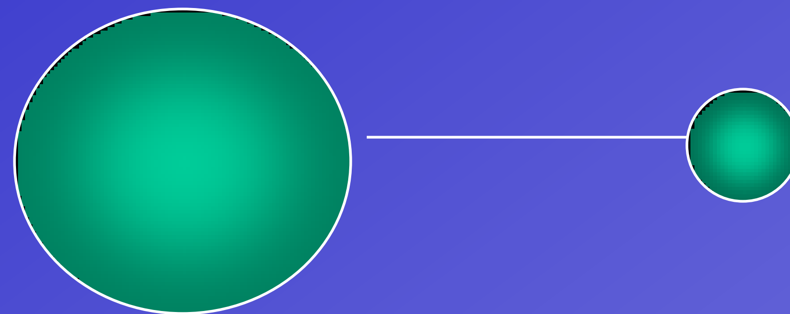
2. The electric field inside the conductor is zero

3. The charge is only distributed on its surface

4. Its self is an equipotential.

5.the field at the surface is normal to the surface

## Charge sharing



Two spherical conductor with radius R and r is connected by a wire and charged with Q

$$U_1 = \frac{q_1}{4\pi\varepsilon_0 R} \qquad U_2 = \frac{q_2}{4\pi\varepsilon_0 r}$$

$$U_1 = U_2$$

$$q_1 = \frac{QR}{R+r} \qquad q_2 = \frac{Qr}{R+r}$$

$$\sigma_1 = \frac{Q}{4\pi R(R+r)} \qquad \sigma_2 = \frac{Q}{4\pi r(R+r)}$$

The electric field near a conductor is larger near regions of sharp curvature

Even lightning rod can be fashionable for a time

# Electric shield(静电屏蔽)