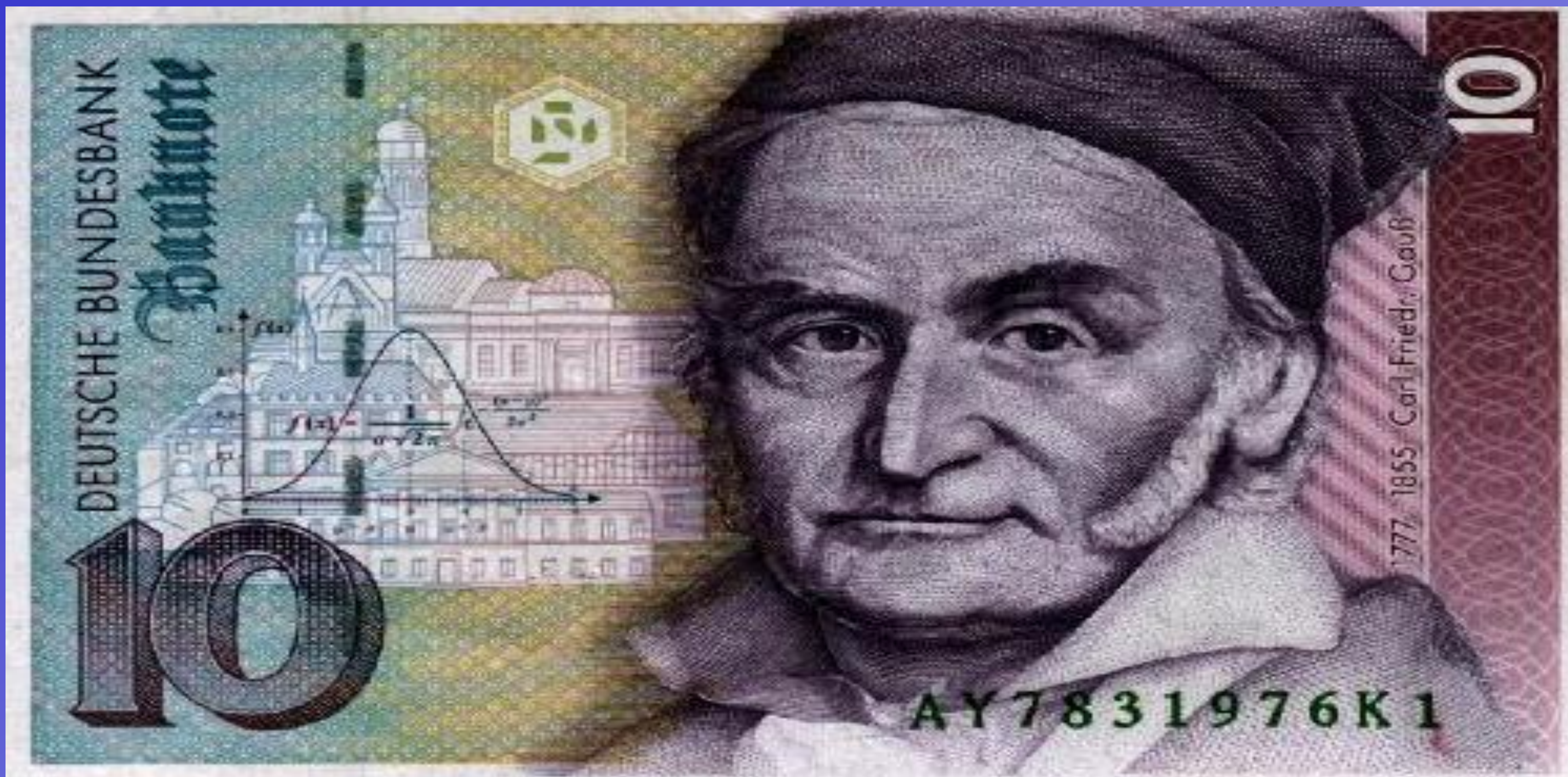
Gauss’s Law(高斯定律）



Agenda today

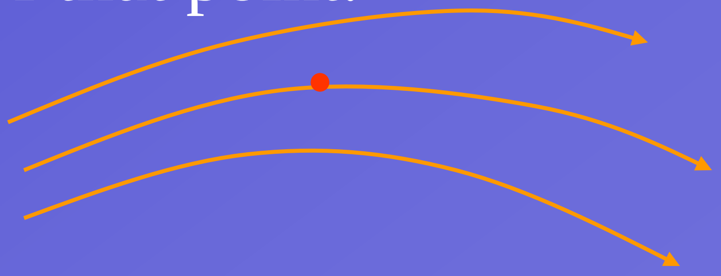
1. Electric field lines
2. Flux of electric field
3. Gauss’s law
4. Application of Gauss’s law

Electric field lines(电力线)

The rules for drawing electric field line:

1. Electric field lines are drawn so that the tangent to the field line at each point specifies the direction of the electric field E at that point.

2.The density in space of electric field line around a particular point is proportional to the strength of electric field of that point.



Properties of electric field line:

1. Electric field lines originate on positive charges, terminate on negative charges.

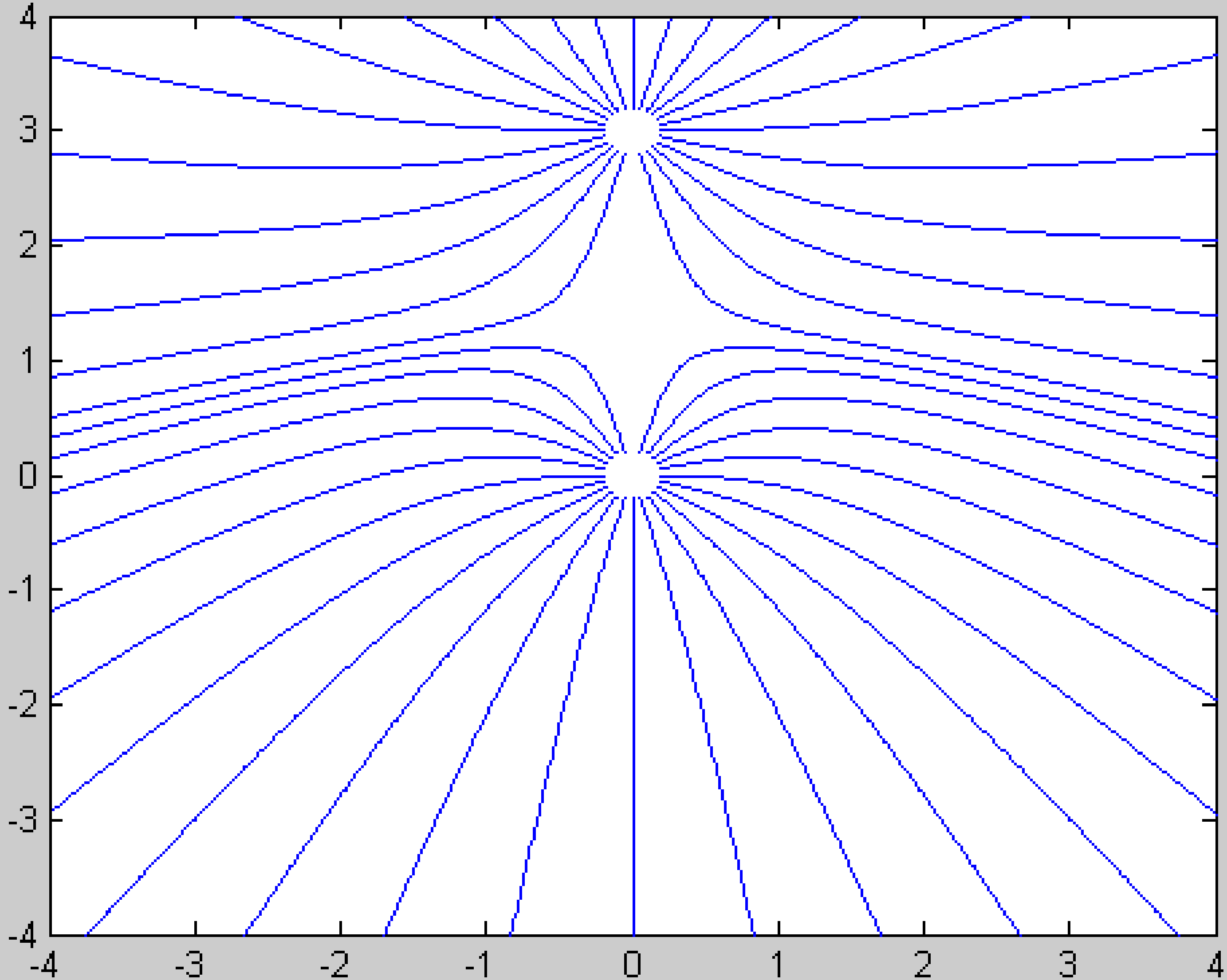
2.no two field line ever cross each other.

Sometimes symmetry is quite useful to determine the electric field line

Example: Draw the electric field lines for a system that consists of two charges , +2q and -q

```
for j=-1*pi:pi/12:pi
    y(1)=0.2*sin(j);
    x(1)=0.2*cos(j);
for i=1:400
r(i)=sqrt(x(i)^2+y(i)^2);
r1(i)=sqrt(x(i)^2+(3-y(i))^2);
x1(i)=x(i)/(r(i)^3)+2*x(i)/(r1(i)^3);
y1(i)=y(i)/(r(i)^3)-2*(3-y(i))/(r1(i)^3);
y(i+1)=y(i)+y1(i)/x1(i)*0.01;
x(i+1)=x(i)+0.01;
```

```
if x(1)<=0
    x(i+1)=x(i)-0.01;
    y(i+1)=y(i)-y1(i)/x1(i)*0.01;
end
end
if (abs(x(1))>0.0001) | (y(1)<0)
plot(x,y);
axis([-4 4 -4 4]);
hold on
end
end
```


4
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2
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0
-1
-2
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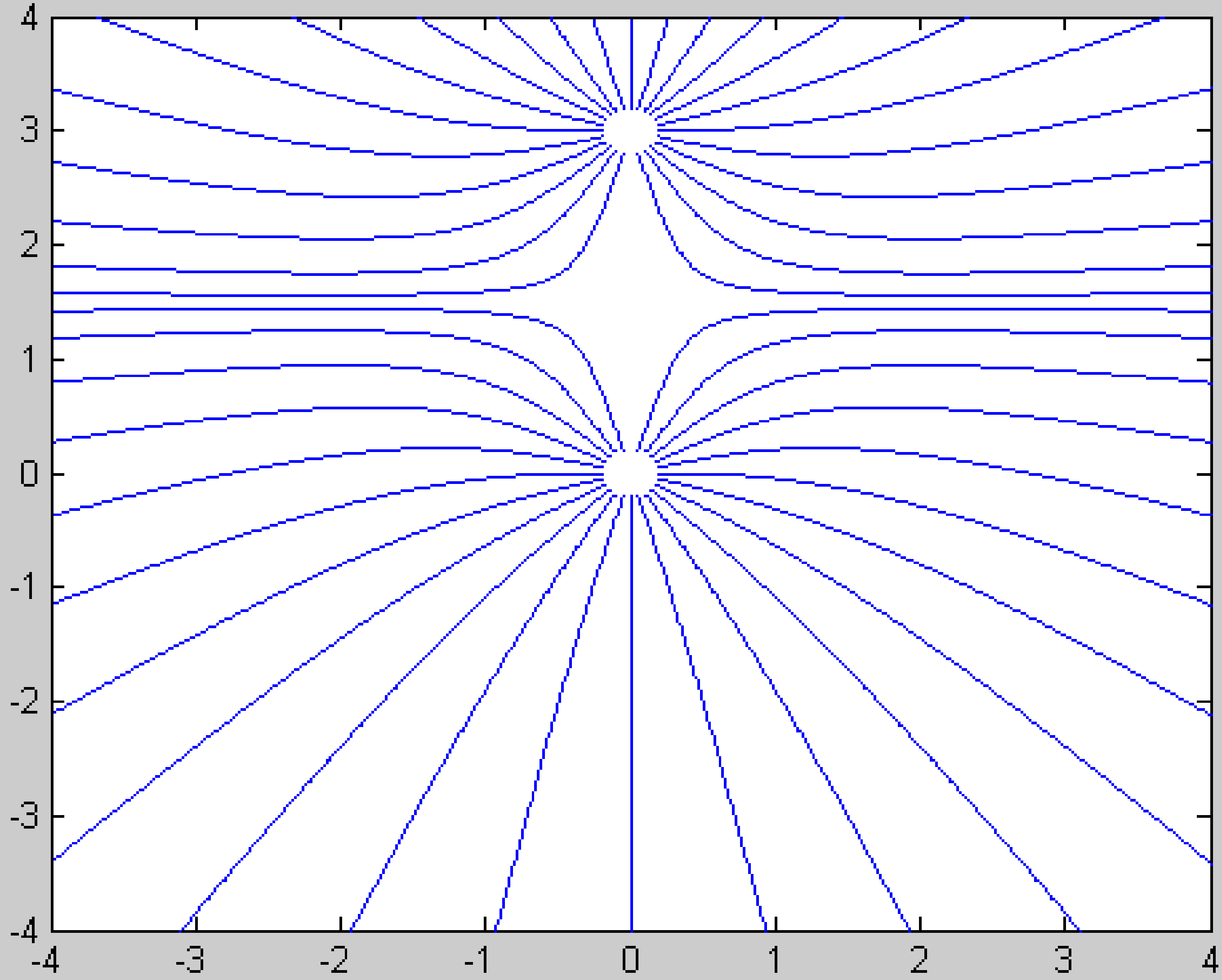
1
2
3
4


4
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-1
0
1
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3
4

Flux(通量):

$$\Phi_e = \int_S E\cos\theta\, d\theta = \int_S \vec{E}\cdot d\vec{S}$$

For a closed surface(Gauss surface)

$$\boxed{\Phi_e = \oint_S \vec{E}\cdot d\vec{S}}$$

Electric flux through a surface is proportional to the number of electric field lines that pass through the surface

Example:

A non-uniform electric field given as $\mathbf{E} = 3.0x\mathbf{i} + 4.0\mathbf{j}$ pierces the Gauss Surface as the figure shows . What is the electric flux through the right surface and top surface?

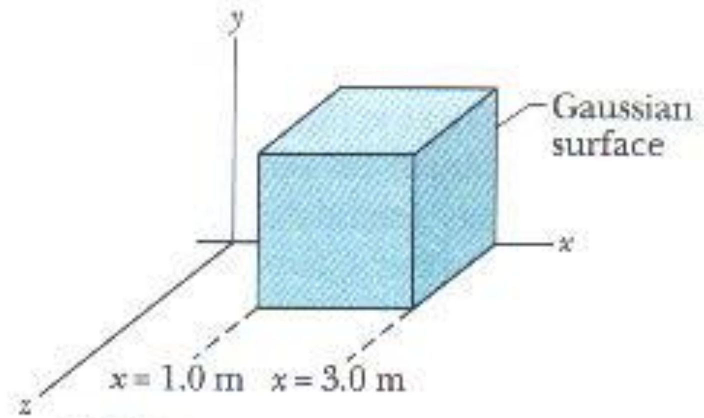


FIGURE 24-5 Sample Problem 24-2. A Gaussian cube with one edge on the x axis lies within a nonuniform electric field.

Solution:

right surface: $d\mathbf{A} = dA\mathbf{i}$

[illegible]

Top surface: $d\mathbf{A} = dA\mathbf{j}$

[illegible]

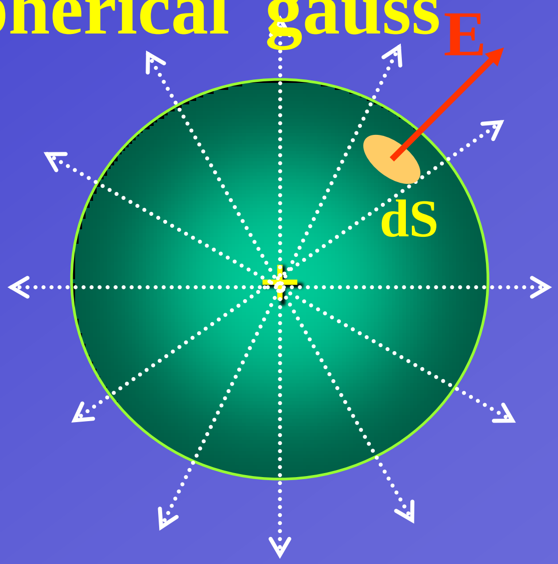
Gauss's Law:

The electric flux in free space through any closed surface equals to the sum of charge enclosed in the surface divided by the permittivity of free space.

$$\Phi_e = \oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_o} \sum_{i=1}^{n} q_i$$

Verification for Gauss's Law

A point charge at the center of spherical gauss surface

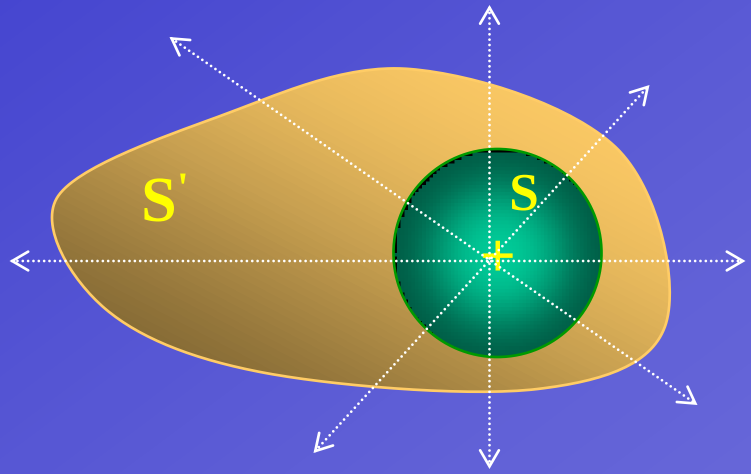


$$E = \frac{q}{4\pi\varepsilon_o R^2}$$

$$d\Phi_e = E\cos 0^\circ dS = \frac{q \cdot dS}{4\pi\varepsilon_o R^2}$$

$$\Phi_e = \oint_S \frac{qdS}{4\pi\varepsilon_o R^2} = \frac{q}{4\pi\varepsilon_o R^2} \cdot 4\pi R^2 = \frac{q}{\varepsilon_o}$$

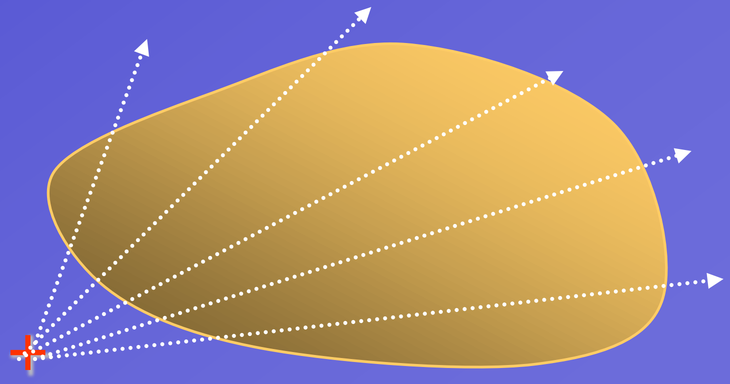
For non-spherical Gauss surface



For many point charges

$$\Phi_e = \oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_o} \sum_{i=1}^{n} q_i$$

For charges outside of the Gauss surface



$$\Phi_e = \oint_S \vec{E} \cdot d\vec{S} = 0$$

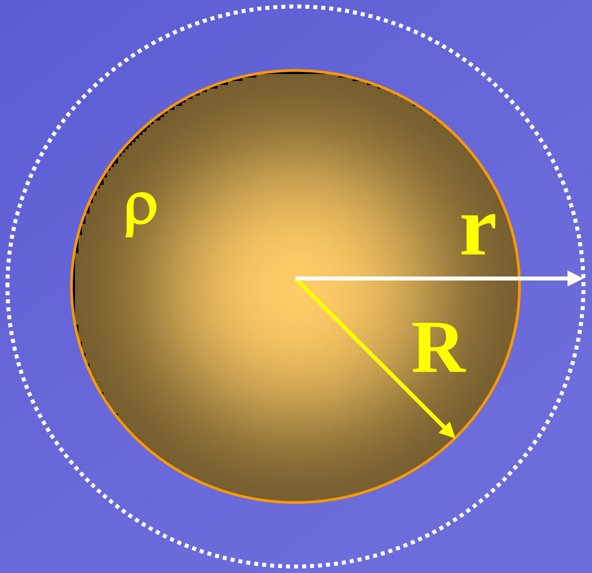
Application of Gauss's Law:

Electric field of charges of spherical symmetry :

The sphere's radius is R, the charge density of it is ρ .

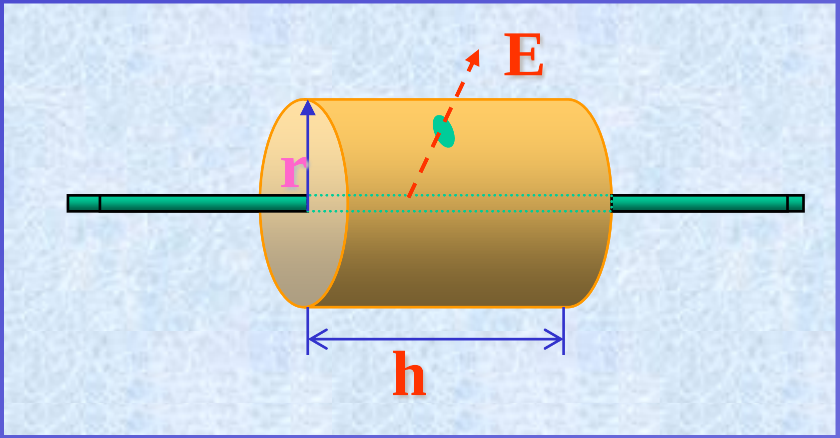
$$E = \frac{\rho R^3}{3\varepsilon_0 r^2} \qquad (r \geq R)$$

$$E = \frac{\rho r}{3\varepsilon_o} \qquad (r < R)$$



The electric field of charges of cylindrical symmetry

An infinitely long rod with charges distributed uniformly



Its charge density is λ

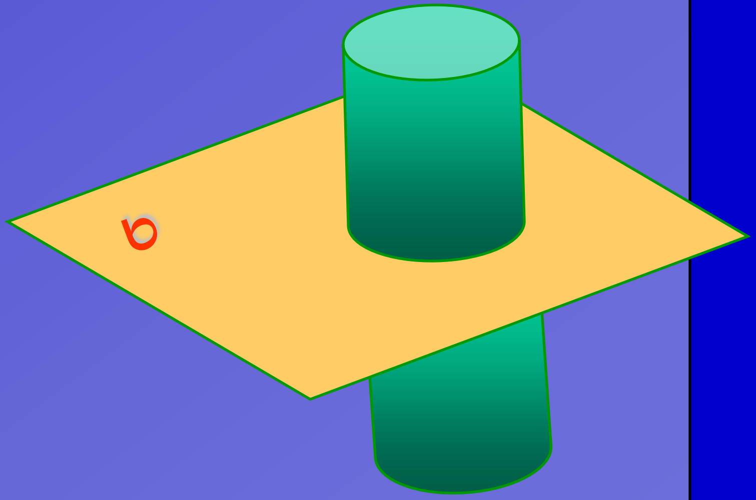
$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

Plane with uniform charge density δ

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{\sigma\, S}{\varepsilon_o}$$

$$2ES = \frac{\sigma\, S}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

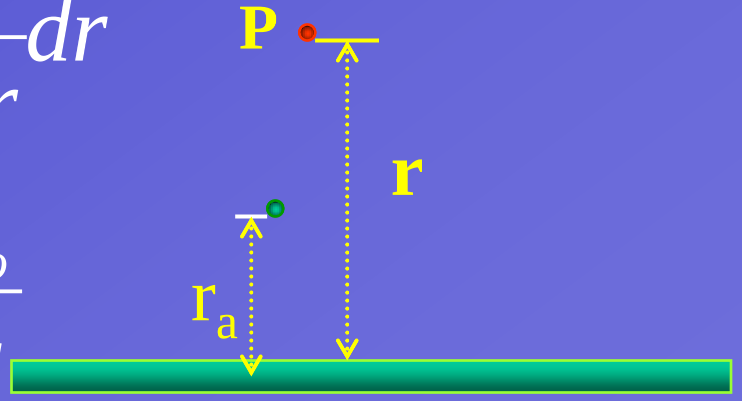


Example: The charged line. Find the electric potential as a function of the radial distance R from an infinite charged line of uniform charge density λ.

Solution: $$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

$$\Delta U = \int_r^{r_o} \vec{E}\cdot d\vec{l} = \int_r^{r_o} \frac{\lambda}{2\pi\varepsilon_o r} dr$$

$$= \frac{\lambda}{2\pi\varepsilon_o} \ln r \Big|_r^{r_o} = \frac{\lambda}{2\pi\varepsilon_o} \ln \frac{r_o}{r}$$



Let zero potential be at r=a, so we have:

$$U_r = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{a}{r}$$