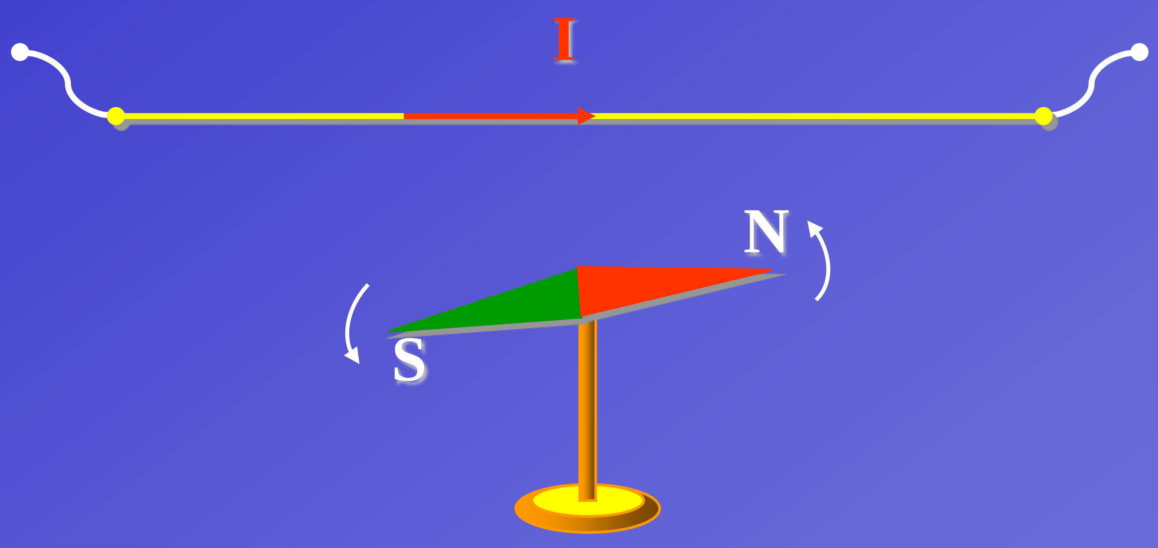
Source of magnetic field

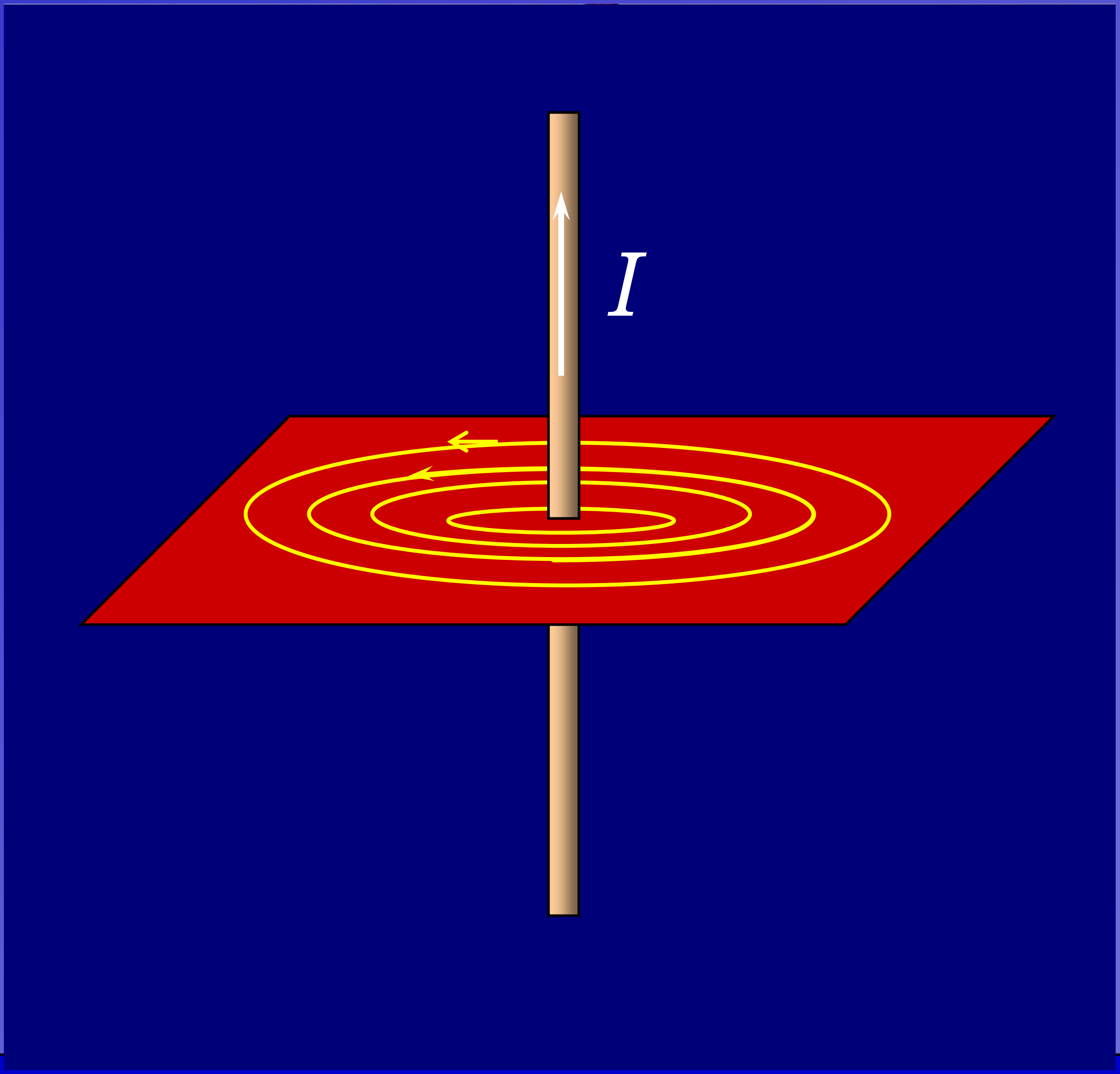
Agenda today

1. Biot-Savart-Laplace Law
2. the application of Biot-Savart-Laplace Law
3. Ampere’s Law
4. The magnetic field of toroid and solenoid

The source of magnetic field



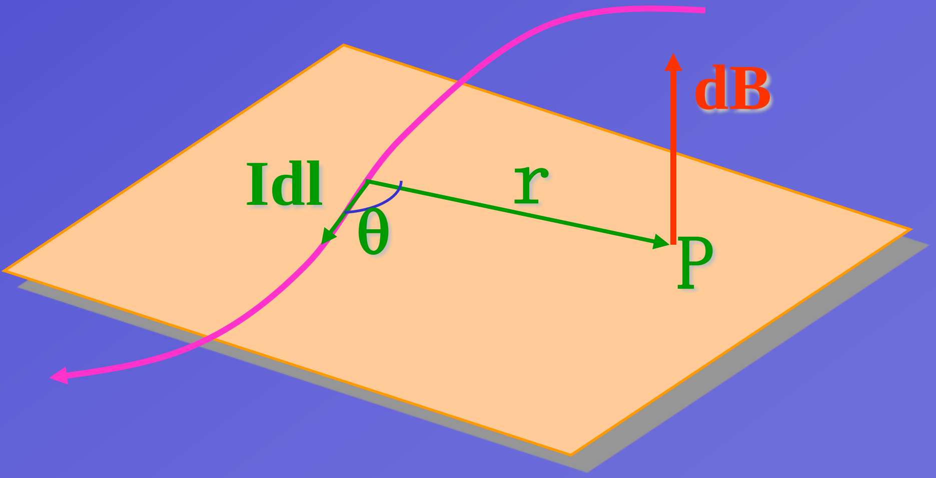
The Magnetic field of DC current along a straight wire



Biot-Savart-Laplace Law(毕奥－萨伐尔－拉普拉斯定律）

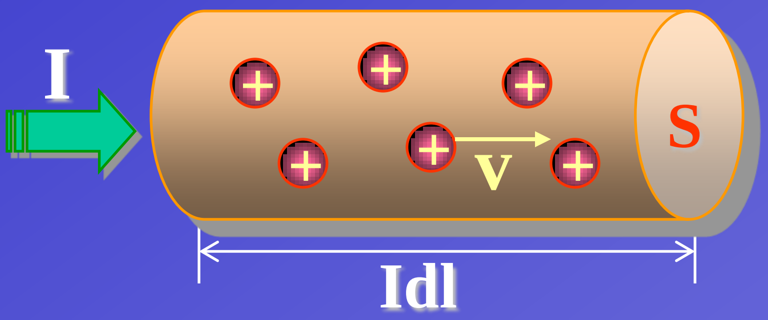
$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

Where $\mu_0 = 4\pi * 10^{-7}$ (T*m/A) is the permeability of free space（真空磁导率）



The magnetic field of moving charge

$$I = nqvs$$



$$d\vec{B} = \frac{\mu_o I d\vec{l} \times \hat{r}}{4\pi r^2}$$

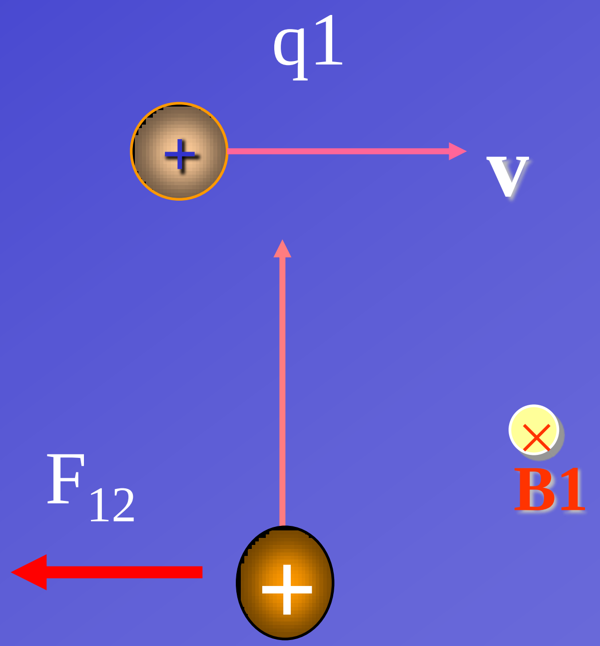
$$dN = nSdl$$

$$d\vec{B} = \frac{\mu_o\, nqdlS\ \vec{v} \times \hat{r}}{4\pi r^2}$$

$$d\vec{B} = \frac{\mu_o\, dN\ q\vec{v} \times \hat{r}}{4\pi r^2}$$

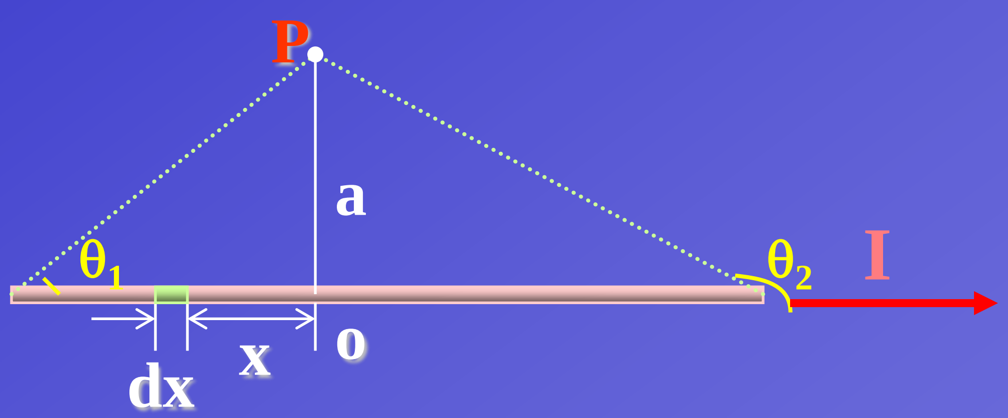
$$\vec{B} = \frac{\mu_o}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

The magnetic force and conservation of momentum



The application of Biot-Savart Law

B due to Current in a straight wire



$$dB = \frac{\mu_o}{4\pi}\frac{Idx\sin\theta}{r^2}$$

$$x = -a\,ctg\theta$$

$$dx = \frac{a\,d\theta}{\sin^2\theta}$$

$$r = \frac{a}{\sin\theta}$$

$$B = \int dB = \int \frac{\mu_o I}{4\pi} \frac{a}{\sin^2\theta} \frac{\sin^2\theta}{a^2} \sin\theta\, d\theta$$

$$B = \frac{\mu_o I}{4\pi\, a} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$B = \frac{\mu_o I}{4\pi a}\left(\cos\theta_1 - \cos\theta_2\right)$$

If the wire is infinite long

$\theta_1 = 0$, $\theta_2 = \pi$

$$B = \frac{\mu_o I}{2\pi a}$$

The definition of the Ampere

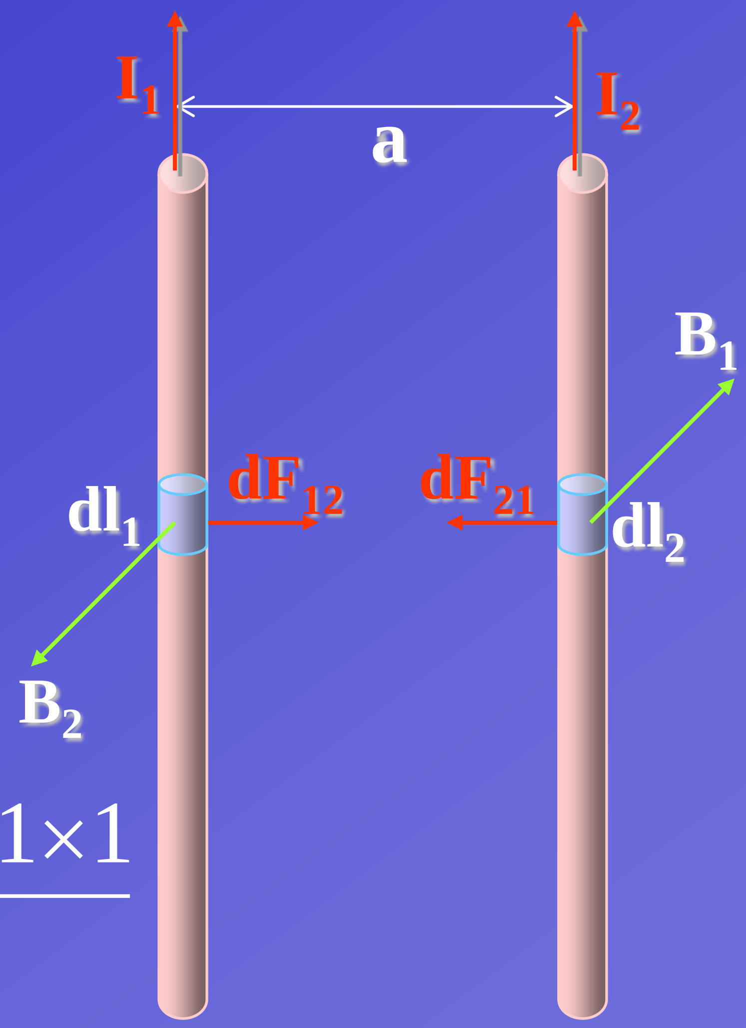
If two very long parallel wires one meter apart carry equal current in each is defined to be one ampere if the force per unit length on each wire is $2*10^{-7}$ N/m

$$B_2 = \frac{\mu_o I_2}{2\pi a} \qquad B_1 = \frac{\mu_o I_1}{2\pi a}$$

$$dF_{12} = I_1 B_2 dl_1 = \frac{\mu_o I_1 I_2}{2\pi a} dl_1$$

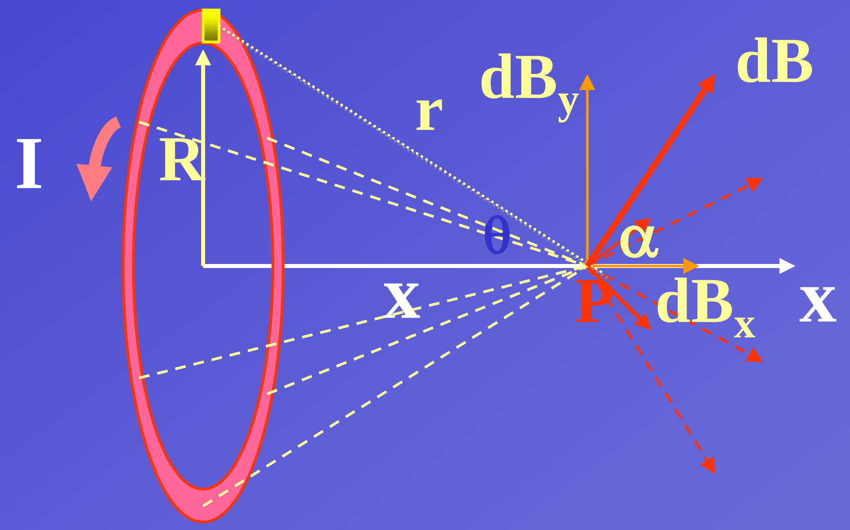
$$\frac{dF}{dl} = \frac{\mu_o I_1 I_2}{2\pi a} = \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 1}$$

$$= 2 \times 10^{-7}\, N \cdot m^{-1}$$



B due to current in circular loop

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2}$$



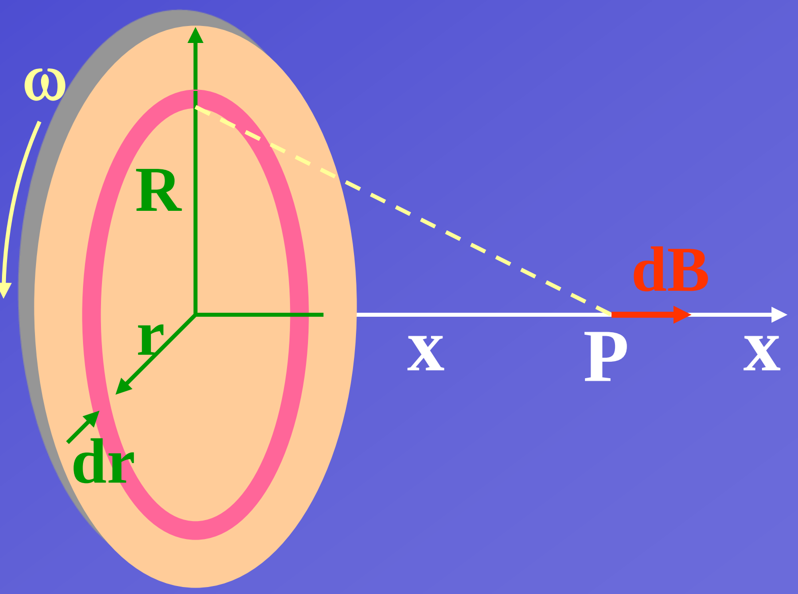
$$\because \quad \vec{dl} \perp \vec{r}, \therefore \quad \theta = 90^\circ$$

$$B = B_x = \int dB\cos\alpha = \int \frac{\mu_o}{4\pi}\frac{Idl\sin 90^\circ\cos\alpha}{r^2}$$

$$B = \int_0^{2\pi R}\frac{\mu_o}{4\pi}\frac{Rdl}{\left(R^2+x^2\right)^{3/2}} = \frac{\mu_0 IR^2}{2\left(R^2+x^2\right)^{3/2}}$$

A disk with radius R is uniformly charged with charge density σ, the disk is rotating about the axis through its center with angular velocity ω, Find the magnetic field along the axis of the disk, and the magnetic moment of the disk

solution



$$dB = \frac{\mu_o r^2 dI}{2(x^2 + r^2)^{3/2}}$$

$$dI = \frac{\omega}{2\pi} dq$$

$$dq = \sigma \cdot 2\pi\, r dr$$

$$dI = \omega\,\sigma\, r dr$$

$$B = \int dB = \int_0^R \frac{\mu_o r^3 \omega \sigma \, dr}{2(x^2 + r^2)^{3/2}}$$

$$= \frac{\mu_o \omega \sigma}{2} \left(\frac{R^2 + 2x^2}{\sqrt{R^2 + x^2}} - 2x \right)$$

Magnetic moment

$$dp_m = \pi r^2 dI = \pi r^2 \omega \sigma r dr = \pi r^3 \omega \sigma \, dr$$

$$p_m = \int_0^R \pi r^3 \omega \sigma \, dr = \frac{1}{4} \pi \omega \sigma R^4$$

Ampere's Law（安培定律）

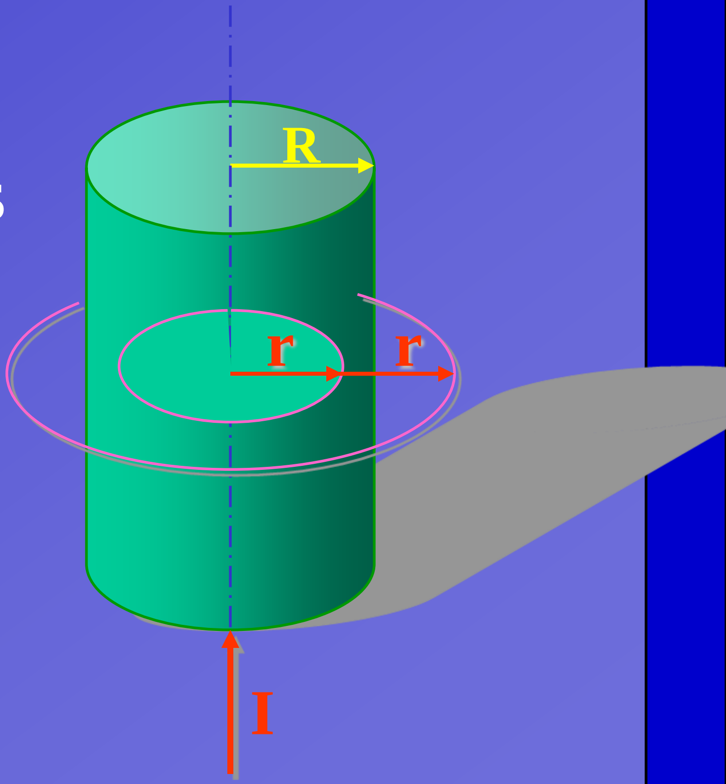
$$\oint_L \vec{B} \cdot d\vec{l} = \mu_o I_c$$

Where I_c stands for the net current that penetrates the area bounded by the curve L

If integral path and current obey right hand law, the current is a positive one, vice versa.

Only the symmetry is enough, can we use ampere's law to find the magnetic field

Example: A long, straight wire of radius a carries a current I that is uniformly distributed over the cross section area of the wire. Find the magnetic field both inside and outside the wire



For magnetic field outside the wire

$$\oint_L \vec{B}\cdot d\vec{l} = B\cdot 2\pi r = \mu_o I$$

$$B = \frac{\mu_o I}{2\pi r}$$

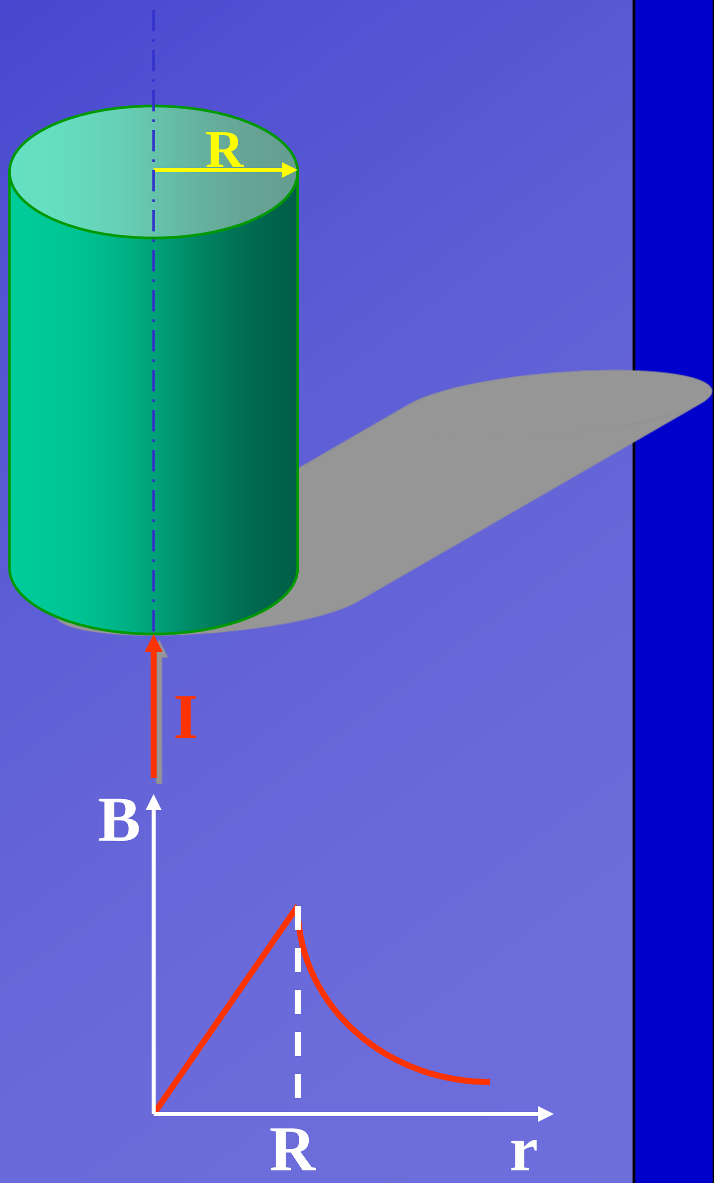
For magnetic field inside the wire

$$I' = \frac{I}{\pi R^2}\cdot \pi r^2 = \frac{r^2}{R^2} I$$

$$\oint_L \vec{B}\cdot d\vec{l} = B\cdot 2\pi r = \mu_o \frac{r^2}{R^2} I$$

$$B = \frac{\mu_o r I}{2\pi R^2}$$

$$B = \frac{\mu_o I}{2\pi r}$$

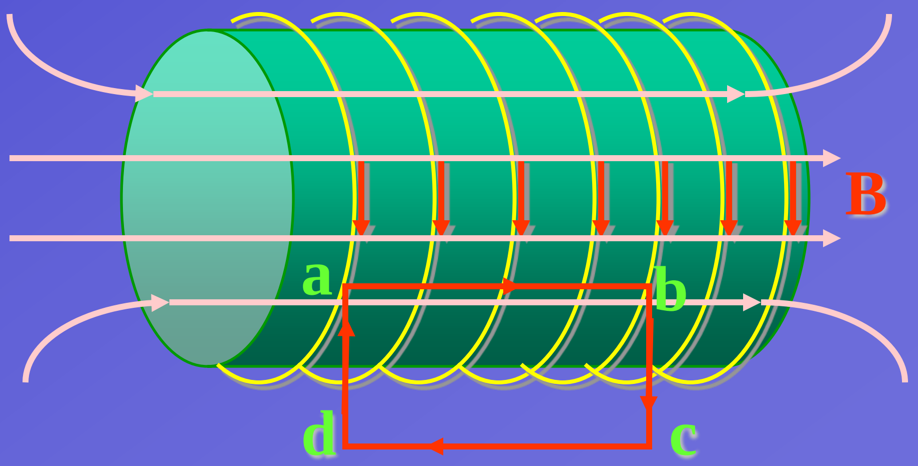


The magnetic field due to current in a solenoid（螺线管）

$$\oint_L \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$$

$$\int_b^c \vec{B}\cdot d\vec{l} = \int_d^a \vec{B}\cdot d\vec{l} = 0$$

$$\int_c^d \vec{B}\cdot d\vec{l} = 0$$



$$\oint_L \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} = B \cdot l$$

$$\sum I_i = I \cdot n \cdot l$$

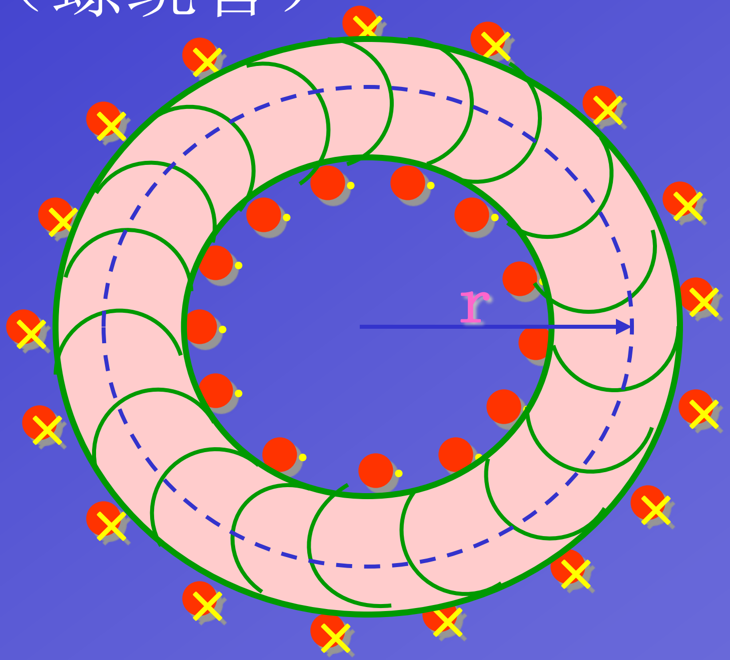
$$\oint_L \vec{B} \cdot d\vec{l} = \mu_o \sum I_i$$

$$B \cdot l = \mu_o I \cdot n \cdot l$$

$$\boxed{B = \mu_o n I}$$

B due to current in tornoid（螺绕管）

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_o \sum I$$



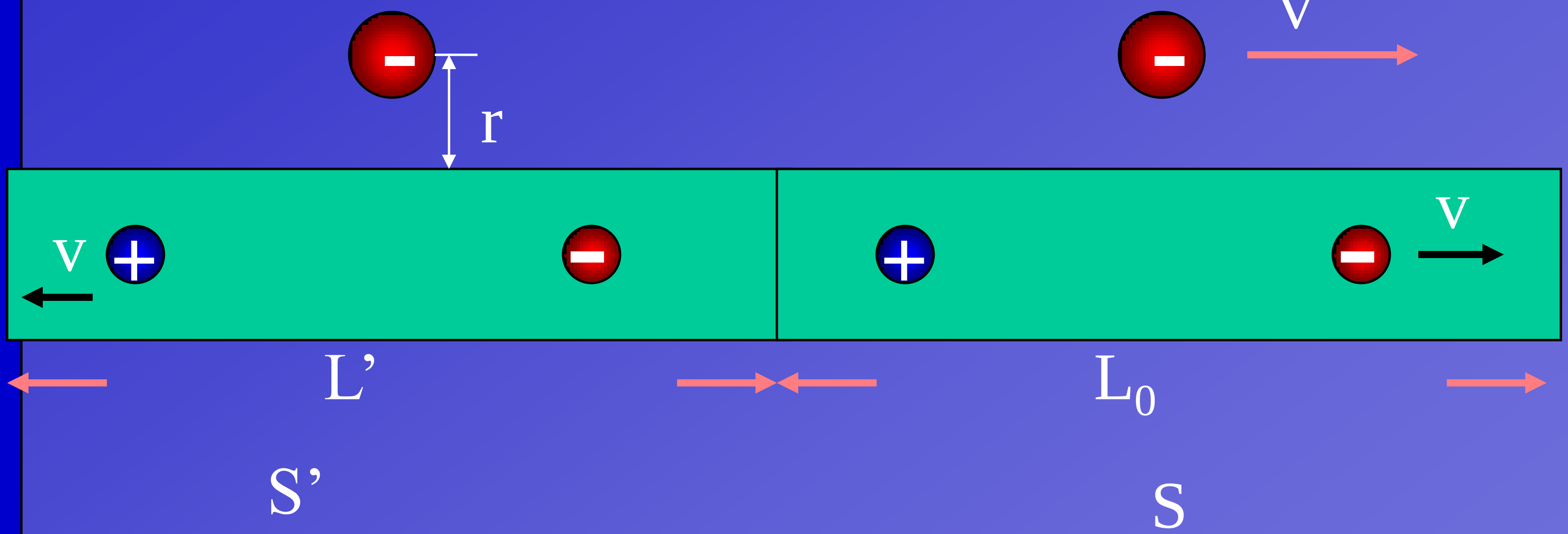
$$B\cdot 2\pi r = \mu_o NI$$

$$B = \frac{\mu_o NI}{2\pi r}$$

$$n = \frac{N}{2\pi r}$$

$$\boxed{B = \mu_o nI}$$

Magnetism : from the viewpoint of relativity



$$\lambda'_+ = \frac{\lambda_+}{\sqrt{1-\beta^2}} \qquad \lambda_- = \frac{\lambda'_-}{\sqrt{1-\beta^2}}$$

$$\lambda_+ + \lambda_- = 0$$

$$\lambda' = \lambda'_+ + \lambda'_- = \lambda_+ \frac{\beta^2}{\sqrt{1-\beta^2}}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\lambda_+ \beta^2}{2\pi\varepsilon_0 r\sqrt{1-\beta^2}}$$

$$F = qE = \frac{qv}{\sqrt{1-\beta^2}}\frac{\lambda v}{2\pi\varepsilon_0 c^2}$$

By Lorenz transformation

$$F' = \sqrt{1-\beta^2}\,F = qv\frac{I}{2\pi d}\frac{1}{\varepsilon_0 c^2} = qv$$

$$B = \frac{\mu_0 I}{2\pi d}$$

$$\therefore c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$