AC circuit


НАРОДНА БАНКА ЈУГОСЛАВИЈЕ
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Agenda Today

1. Self-Inductance and Multi Inductance
2. Energy stored in Magnetic Field
3. RL Circuit
4. AC Circuit

Self-inductance(自感系数)

$$L = \Psi / I$$

Unit: H

According to Faraday's Law:

$$\varepsilon_L = -\frac{d\Psi}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

Self-inductance of ideal solenoid

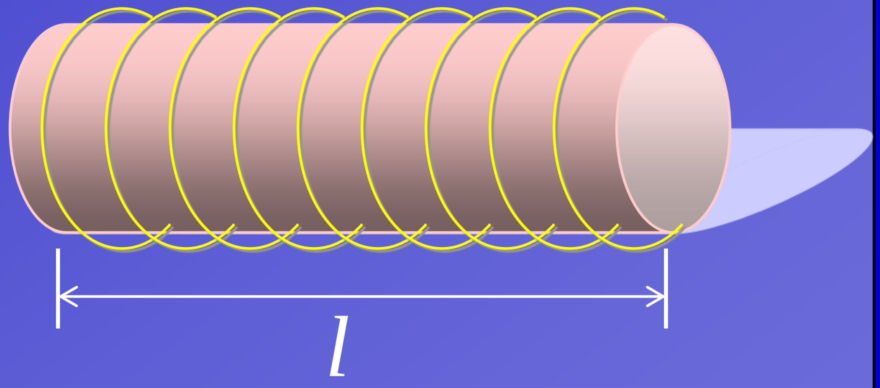
Cross section area : A

length: l

current : I

number of turns: N

permeability: μ



$$B = \mu \frac{N}{l} I$$

$$\Psi = NBS = \mu \frac{N^2}{l} IS$$

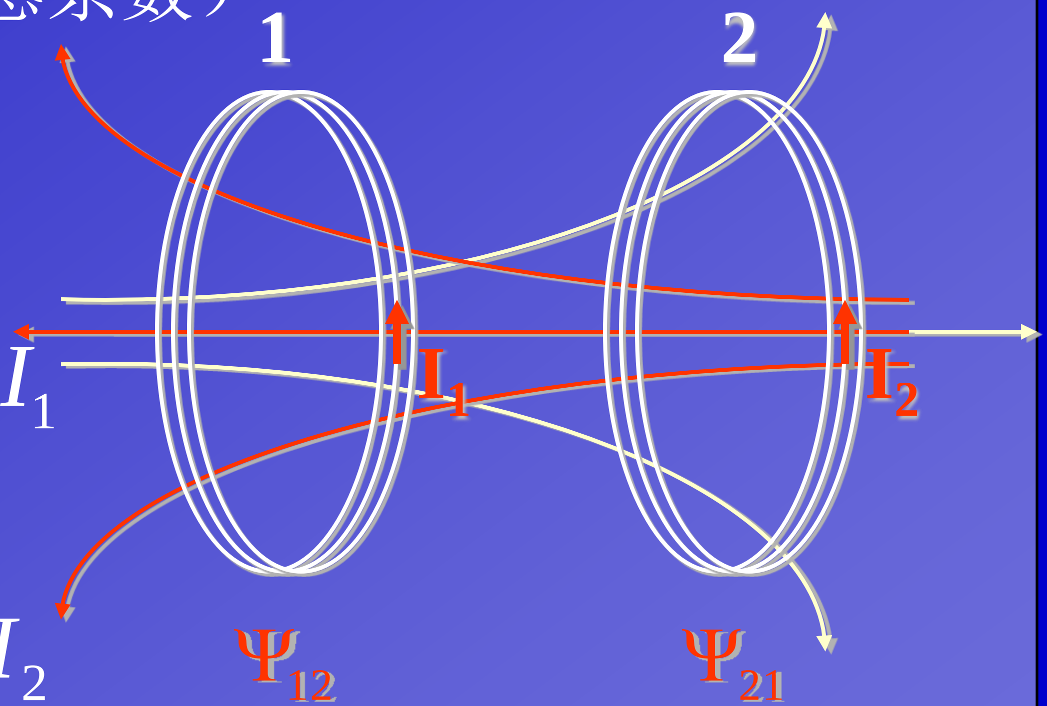
$$L = \frac{\Psi}{I} = \mu \frac{N^2}{l} S = \mu n^2 V$$

Where n is turns per unit length

Multi induction（互感）

Emf caused by the change in current carried by another loop.

Multi inductance（互感系数）



$$\Psi_{21} = N_2\Phi_{21} = M_{21}I_1$$

$$\Psi_{12} = N_1\Phi_{12} = M_{12}I_2$$

$$M_{12} = M_{21} = M$$

$$\varepsilon_{21} = -M\frac{dI_1}{dt}$$

$$\varepsilon_{12} = -M\frac{dI_2}{dt}$$


互感概念演示仪

Tesla coil

Example: Inside the magnetic material with permeability μ is a straight long wire and rectangular loop as the figure shows. find the multi inductance

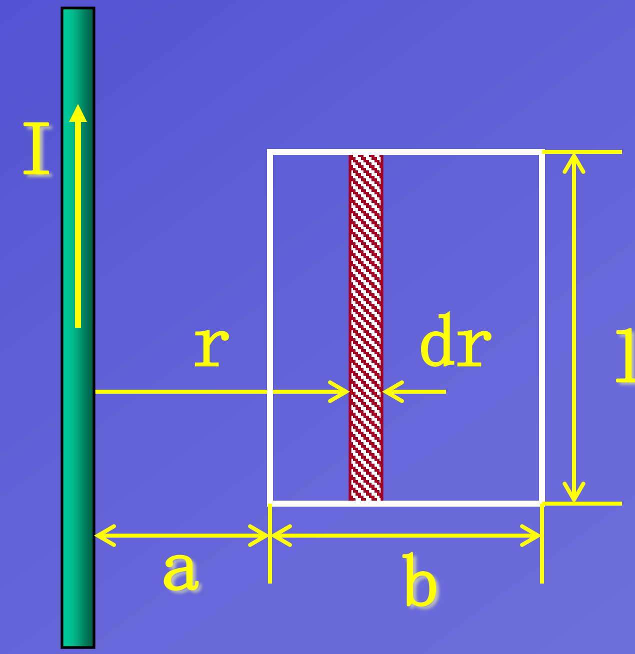
solution

$$B = \frac{\mu_o I}{2\pi r}$$

$$d\Phi = BdS = \frac{\mu_o I}{2\pi r} l dr$$

$$\Phi = \int_a^{a+b} \frac{\mu_o I l}{2\pi r} dr = \frac{\mu_o I l}{2\pi} \ln \frac{a+b}{a}$$

$$M = \frac{\Phi}{I} = \frac{\mu_o l}{2\pi} \ln \frac{a+b}{a}$$



RL circuit

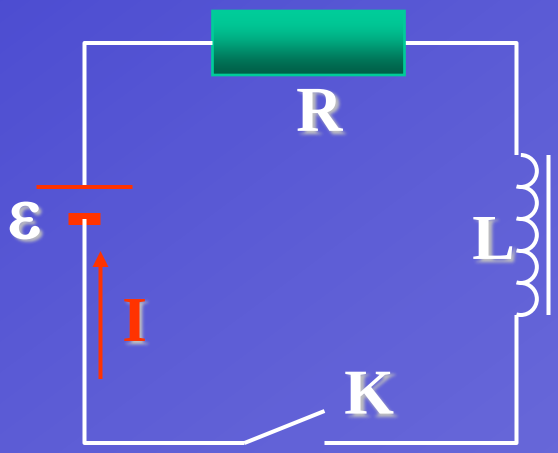
By loop's rule:

$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

$$I\Big|_{t=0} = 0$$

$$I\Big|_{t=\infty} = \frac{\varepsilon}{R}$$

$$I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad \text{where} \quad \tau = \frac{L}{R}$$



Energy stored in inductor

$$dE = UI\,dt = LI\,dI$$

$$E = \int dE = \int_0^{I} LI\,dI = \frac{1}{2}LI^2$$

Energy stored in multi inductors

$$E = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 \pm M I_1 I_2$$

Energy density of magnetic field

For ideal solenoid:

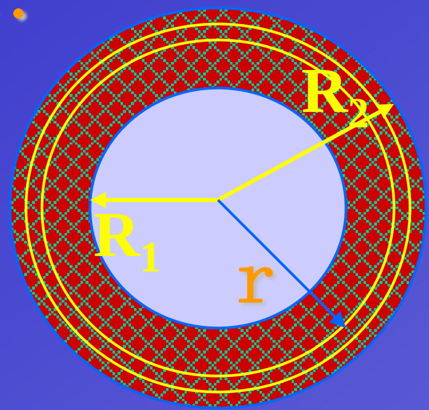
$$L = \mu n^2 V \qquad I = \frac{B}{\mu n}$$

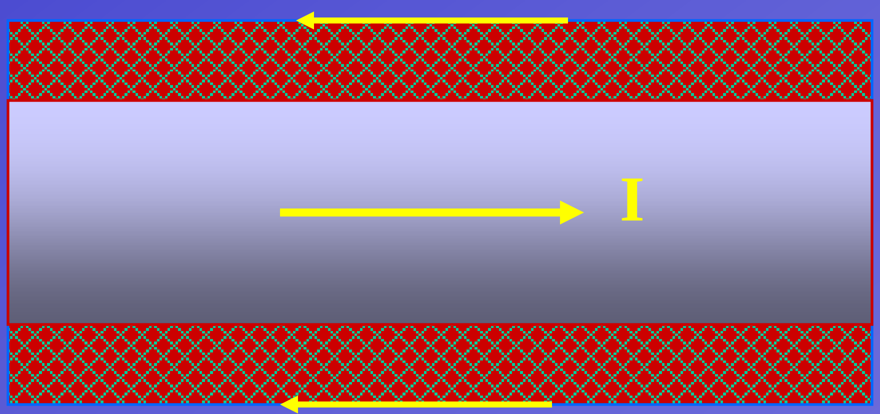
$$W_m = \frac{1}{2} L I^2 = \frac{1}{2} \mu n^2 V \left(\frac{B}{\mu n} \right)^2 = \frac{1}{2} \frac{B^2}{\mu} V$$

$$w_m = \frac{W_m}{V} = \frac{1}{2} \frac{B^2}{\mu} = \frac{1}{2} \mu H^2$$

Example: coaxial cable shown below carries a current of I, Find the energy stored by a segment of the cable with length L.

solution:





$$B = \frac{\mu_o I}{2\pi r} \qquad w_m = \frac{B^2}{2\mu_o} = \frac{\mu_o I}{8\pi^2 r^2} \qquad dV = 2\pi r l dr$$

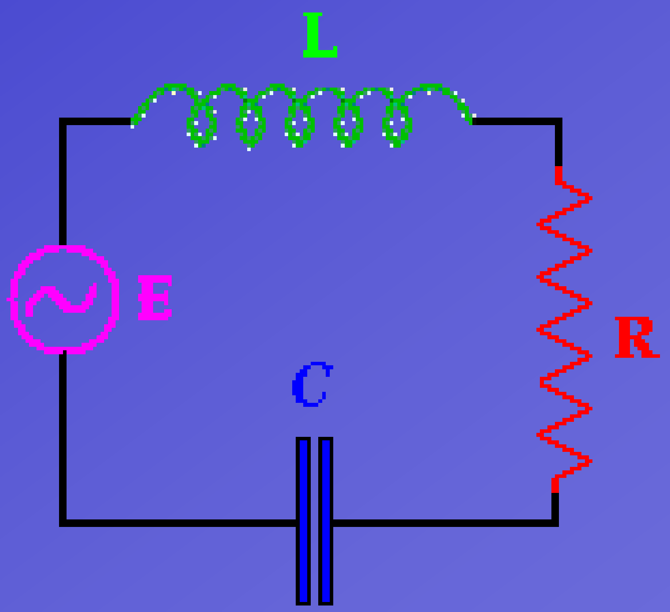
$$W_m = \int_V w_m dV = \int_{R_1}^{R_2} \frac{\mu_o I^2}{8\pi^2 r^2} \cdot 2\pi lrdr$$

$$= \frac{\mu_o I^2 l}{4\pi} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\mu_o I^2 l}{4\pi} \ln \frac{R_2}{R_1}$$

Oscillation in circuit

If E=0, but loop rule

$$L\frac{dI}{dt}+IR+\frac{Q}{C}=0$$



Since: $$I=\frac{dQ}{dt}$$

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

Compare with damped oscillation:

$$m\frac{d^2x}{dt^2}+f\frac{dx}{dt}+kx=0$$

We have the angular frequency of RLC:

$$\omega_0=\frac{1}{\sqrt{LC}}$$

RLC circuit with AC source

Like forced oscillation:

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=V\sin\omega t$$

If R is small, and $\omega=\omega_0$

The magnitude of current has its maximum value, which is called electromagnetic resonance

Capacitive circuit

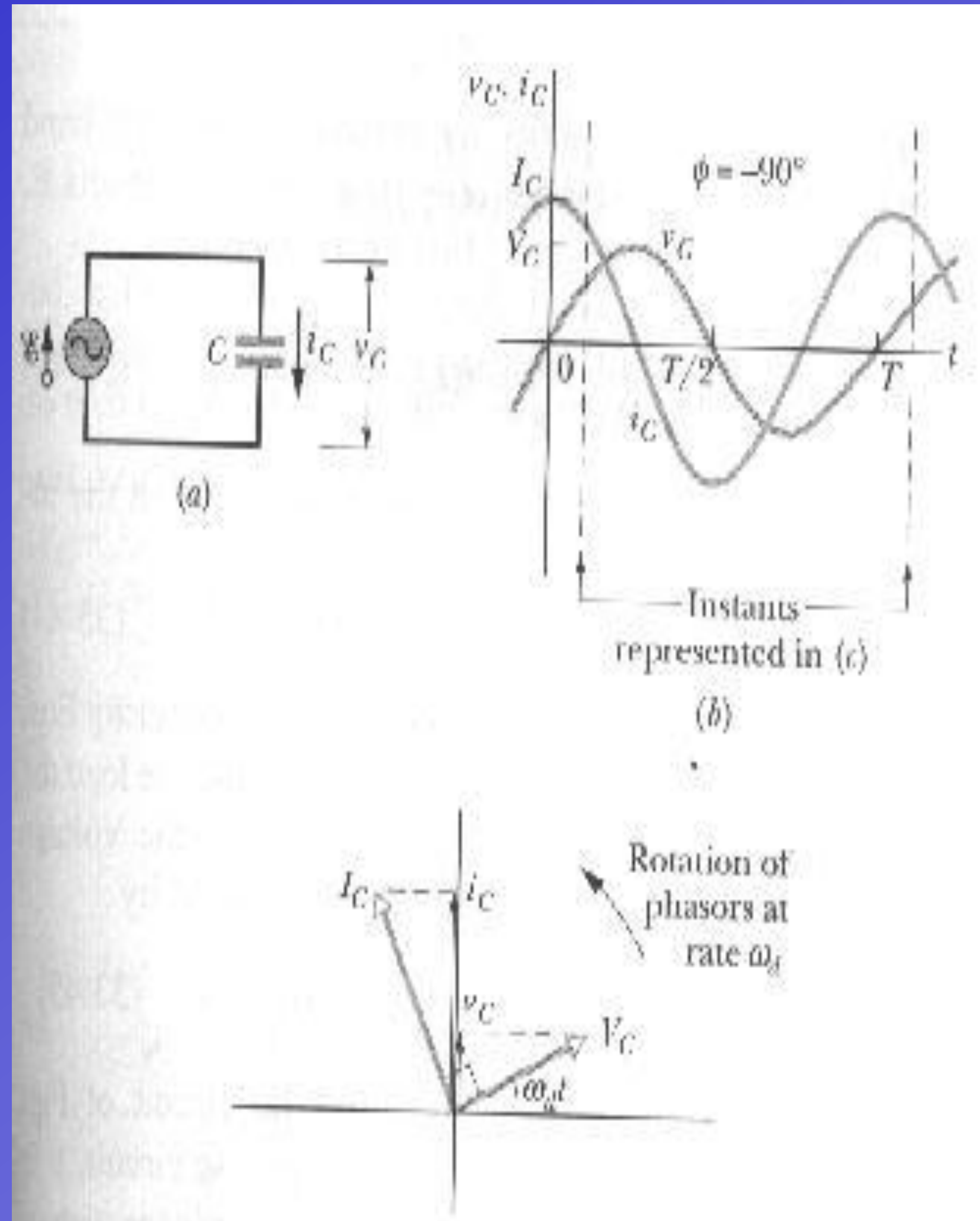
$$V_0 \sin \omega t = \frac{Q}{C}$$

$$I = \frac{dQ}{dt} = \omega C V_0 \cos \omega t$$

$$= \omega C V_0 \sin(\omega t + \frac{\pi}{2})$$

Capacitive reactance

$$X_c = \frac{1}{\omega C}$$



Inductive circuit

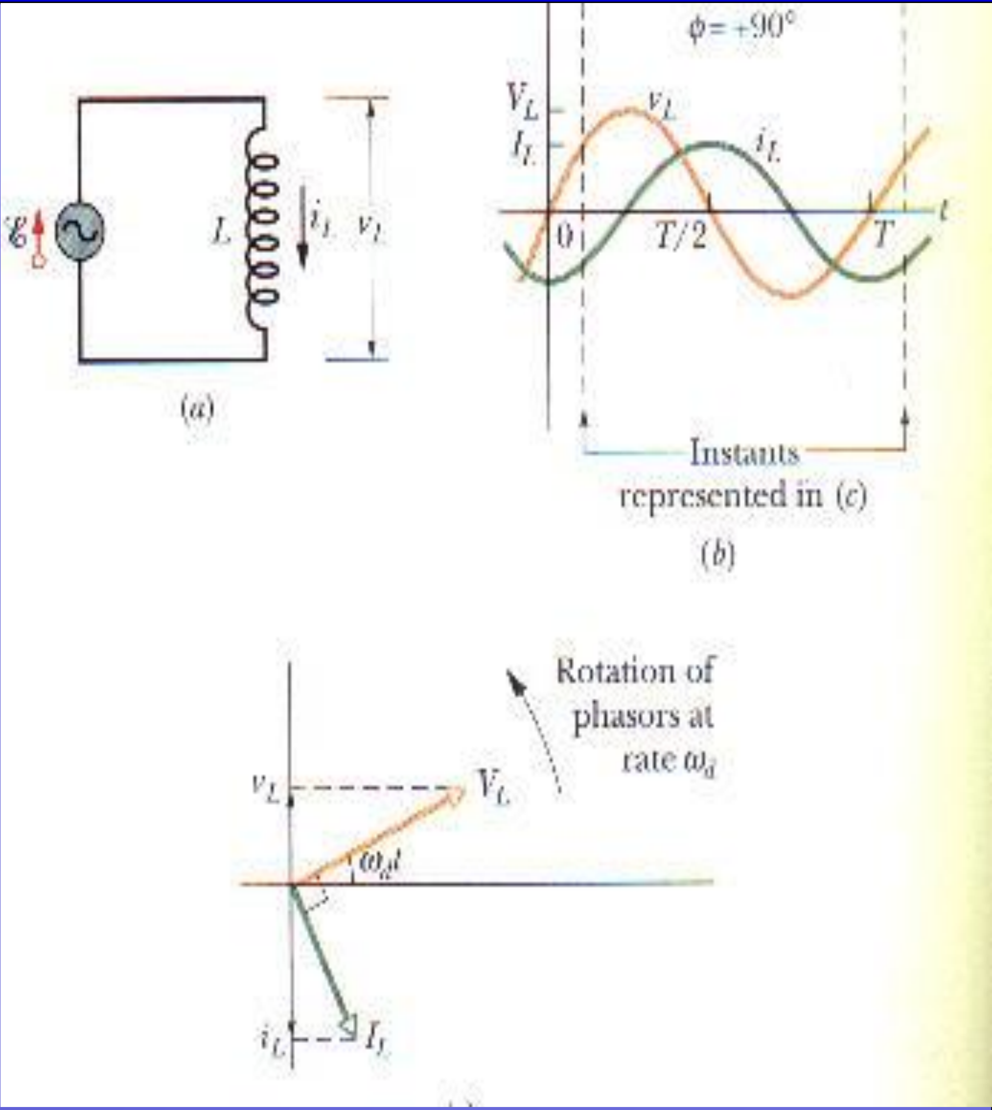
$$V_0 \sin \omega t = L \frac{dI}{dt}$$

$$I = -\frac{V_0}{\omega L} \cos \omega t$$

$$= \frac{V_0}{\omega L} \sin(\omega t - \frac{\pi}{2})$$

Inductive reactance

$$X_L = \omega L$$



Impedance（电抗）

For RLC in series

$$Z = R + i(X_L - X_c)$$

$X_L > X_c$ the circuit is inductive

$X_L < X_c$ the circuit is capacitive

For RLC in parallel

$$\frac{1}{Z_{eff}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots$$

Phasor

[illegible]

[illegible]

where

$$tg\varphi_0 = \frac{X_L - X_C}{R}$$

Ohm’s Law in complex form

V=IZ

Example: The series RLC circuit is driven with an AC source of emf of the form of $V=V_0\sin\omega t$

where $V_0=110$ V, $f=60$Hz, $R=20.0\Omega$, $C=50.0\mu C$,

$L=5.00*10^{-2}$ H, find the potential drops across the inductor, when V reach its maximum value.

Solution:

$$X_L=\omega L=2\pi fL=18.8\Omega$$

$$X_C=\frac{1}{\omega C}=53.0\Omega$$

$$Z=20-i34.2\Omega=39.6\angle-59.6^\circ\ \Omega$$

$$V = 4\sin 6t = 4\cos\left(6t - \frac{\pi}{2}\right) = 4\angle -90^\circ$$

$$I = \frac{V}{Z} = \frac{10\angle -90^\circ}{3.6\angle -56.3^\circ} = 2.8\angle -33.7^\circ \ A$$

$$V_L = [illegible]$$

When V reaches its maximum value

$$\omega t = \frac{\pi}{2}, \quad V_L = [illegible]$$