

# Interference of light waves



# Agenda Today

1. Two-slit experiment
2. Film interference
3. Newton's rings

## Conditions for interference

1 the source should be monochromatic(单色)  
same frequency

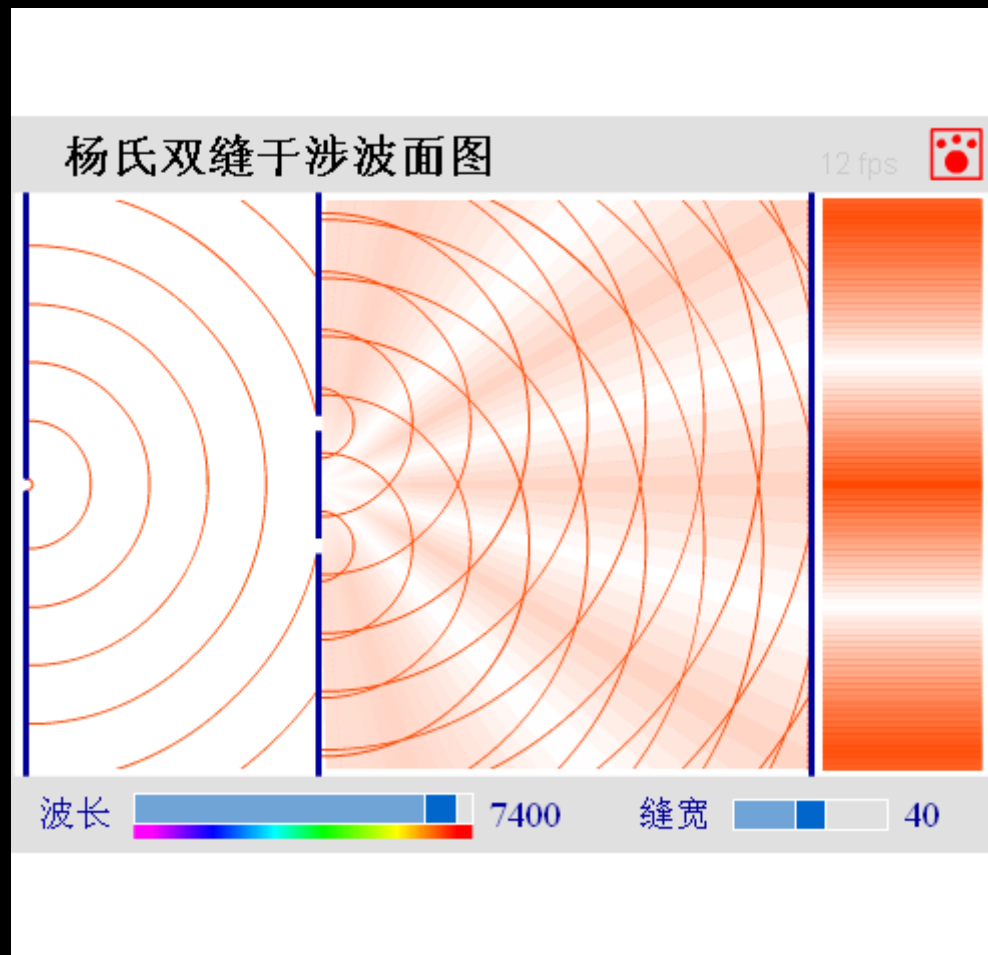
2 the sources must maintain a constant phase  
difference.

Why is there no interference pattern caused by two  
bulbs?

# Young's double-slit experiment (杨氏双缝实验)



Thomas Young (1773-1829)

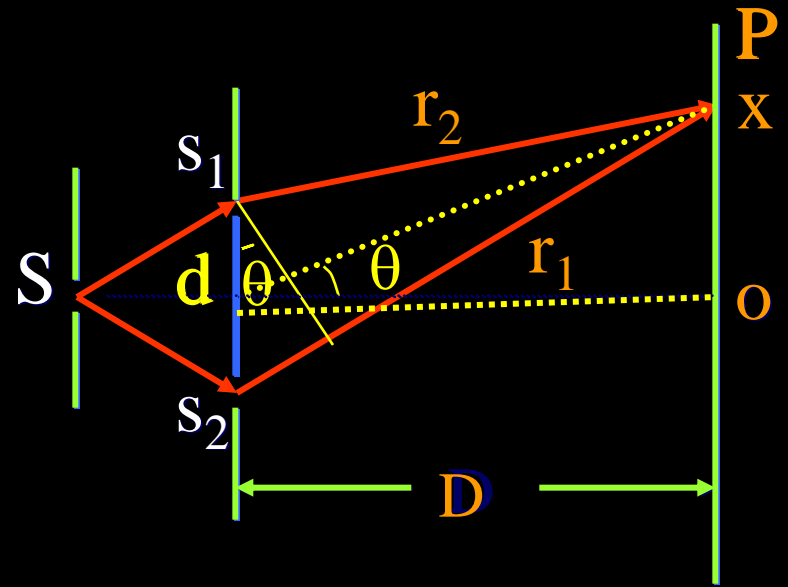


**In the case the double slits:**

$$\delta = r_2 - r_1 \approx d \sin \theta$$

$\because D \gg a$  ,  $\theta$  is very small

$$\therefore \sin \theta \approx \tan \theta = d \frac{x}{D}$$



Constructive

$$d \frac{x}{D} = \pm k \lambda \quad (k = 0, 1, 2, \dots)$$

bright fringes

$$d \frac{x}{D} = \pm (2k - 1) \frac{\lambda}{2} \quad (k = 1, 2, 3, \dots)$$

Destructive

dark fringes

# Intensity distribution of the double-slit interference

$$E_1 = E_0 \cos \alpha$$

$$E_2 = E_0 \cos(\alpha + \Delta\Phi)$$

$$E = E_1 + E_2 = 2E_0 \cos\left(\frac{\Delta\Phi}{2}\right) \cos\left(\alpha + \frac{\Delta\Phi}{2}\right)$$

$$\therefore I \propto E^2$$

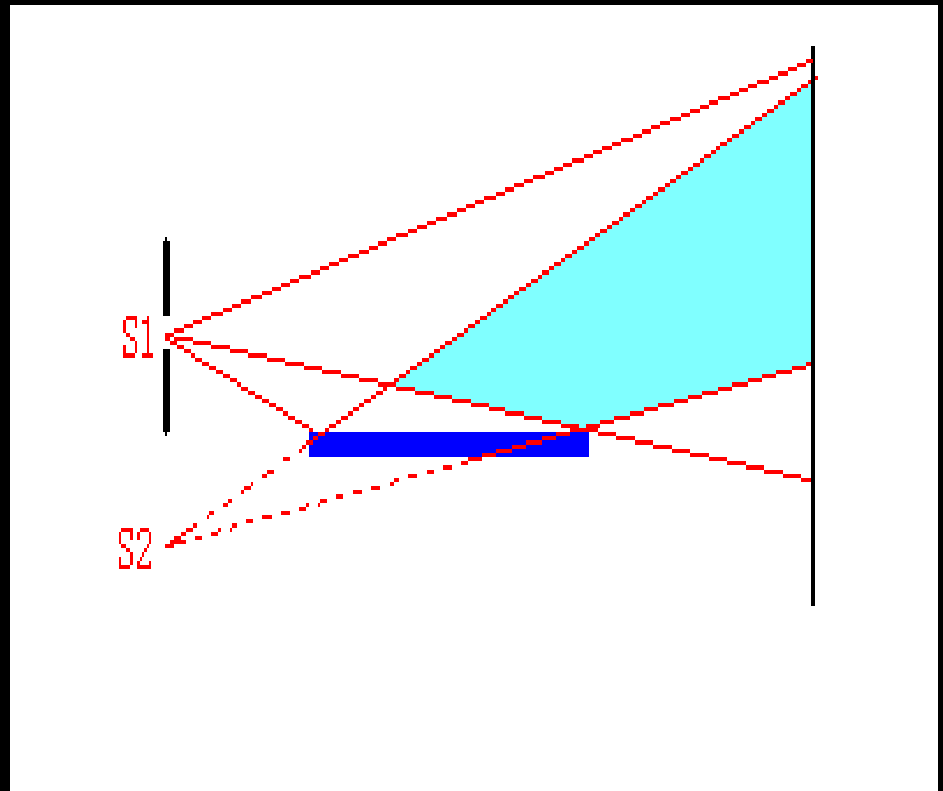
$$\therefore I = I_{\max} \cos^2\left(\frac{\Delta\Phi}{2}\right) = I_{\max} \cos^2\left(\frac{\pi b \sin \theta}{\lambda}\right)$$

**Example:** white light passes through two slits **0.50 mm apart** and an interference pattern is observed on a **screen 2.5 m away**. The first-order fringe resembles a rainbow with violet and red light at either end. The **violet light falls about 2.0 mm** and the **red 3.5 mm**

# Change of phase due to reflection

An electromagnetic wave undergoes a phase change of  $\pi$  upon reflection from a medium that has a higher index of refraction than the one in which the wave is traveling

Lloyd's mirror (洛埃镜) :





## Interference in thin films (薄膜干涉) :

Two things must be noted:

1. The phase shift due to reflection.
2. The wavelength of light  $\lambda_n$  in a medium whose refraction index is  $n$  is

$$\lambda_n = \frac{\lambda}{n}$$

Where  $\lambda$  is the wavelength of the light in free space

# Antireflective coatings(增透膜)



A coating material must have an index of refraction between that of air and the glass to be coated.

$$d_{\min} = \frac{\lambda}{4n}$$

Where  $\lambda$  is the wavelength in vacuum of the rays that we want pass through the coating ,  $n$  is the refraction index.



The colors of this kind of butterfly do not resemble the same pattern on each side of the wings.

**Example:** a certain soap film is in the air ( $n_1=1.33$ ), its thickness is  $0.32\mu\text{m}$ . If a beam of white rays incident on it normally, what is the color of the film when we look downward?

**solution:**  $2n_1e + \frac{\lambda}{2} = k\lambda \rightarrow \lambda = \frac{2n_1e}{k - 1/2}$

$$k = 1, \quad \lambda_1 = \frac{2}{1} n_1 e = 1.70 \mu\text{m} = 17000 \text{ \AA}$$

$$k = 2, \quad \lambda_1 = \frac{4}{3} n_1 e = 0.567 \mu\text{m} = 5670 \text{ \AA}$$

$$k = 3, \quad \lambda_1 = \frac{4}{5} n_1 e = 0.341 \mu\text{m} = 3410 \text{ \AA}$$

$$\lambda_2 = 5670 \text{ \AA} \quad (\text{黄光})$$

Quiz: Two flat glass plates of length  $L=10\text{ cm}$  touch at one end but are separated by a wire of diameter  $d=0.01\text{ mm}$  at the other end, light shines almost perpendicularly on the glass and is reflected into the eyes, what is the distance  $x$  between two bright fringes when  $\lambda=400\text{ nm}$ ?

# Newton's rings

$$2ne + \frac{\lambda}{2} = k\lambda$$

Bright fringes

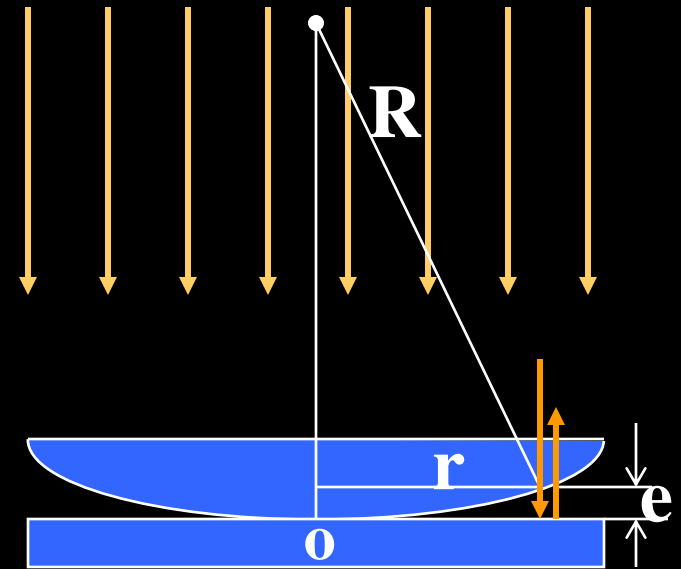
$$2ne + \frac{\lambda}{2} = (2k + 1) \frac{\lambda}{2}$$

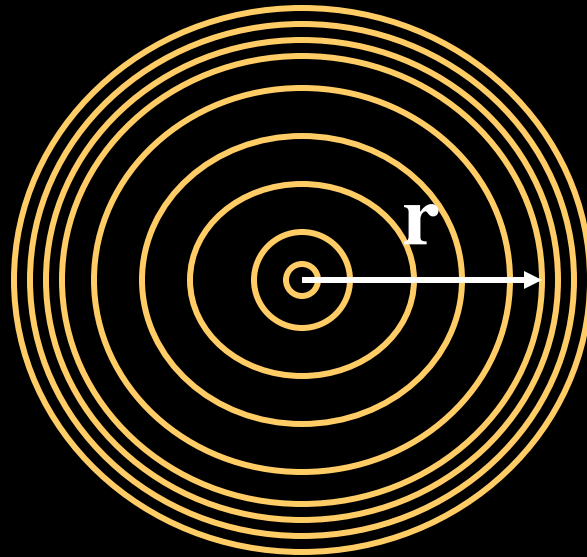
Dark fringes

$$r^2 = R^2 - (R - e)^2 = 2Re - e^2$$

Q  $R \gg e \rightarrow 2Re \gg e^2$

$$e = \frac{r^2}{2R}$$





$$r = \sqrt{\frac{(2k-1)R\lambda}{2n}}$$

$(k = 1, 2, \dots)$

**Bright  
rings**

$$r = \sqrt{\frac{kR\lambda}{n}}$$

$(k = 0, 1, 2, \dots)$

**Dark  
rings**

A certain lamp is used ( $\lambda = 5893\text{\AA}$ ) to observe Newton's rings, the radius of dark ring of  $k$ th order is  $r = 4\text{mm}$ , while that of  $k+5$ th order is  $r = 6\text{mm}$ , find the curvature of the lens.

**solution :**

$$r_k = \sqrt{k\lambda R}$$

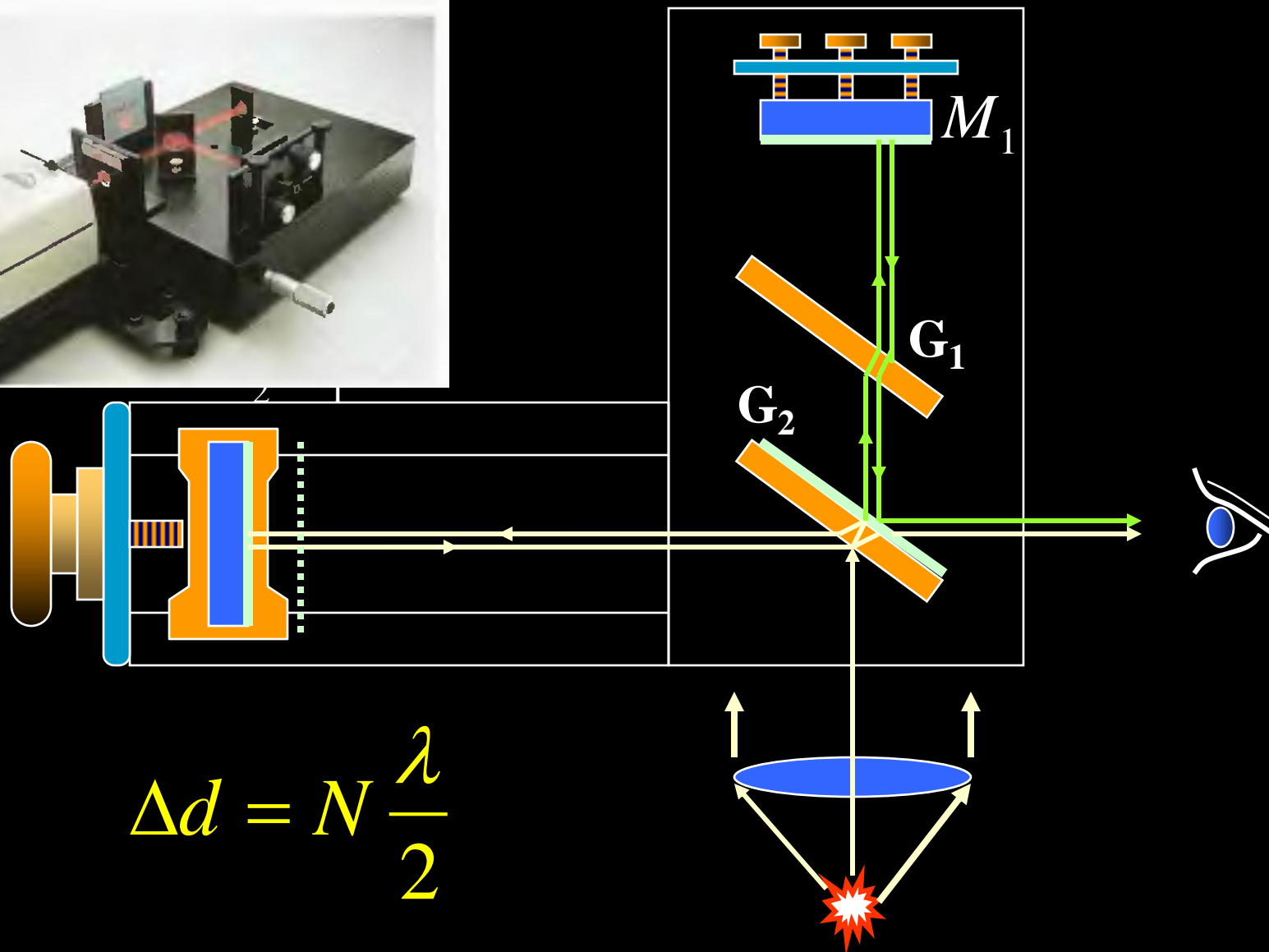
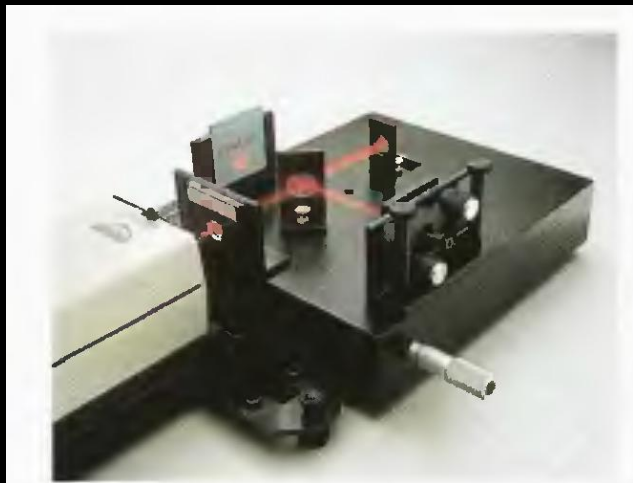
$$r_{k+5} = \sqrt{(k+5)\lambda R}$$

By solving the equations , we will find

$$k = 4 \qquad R = 6.79 \text{ m}$$

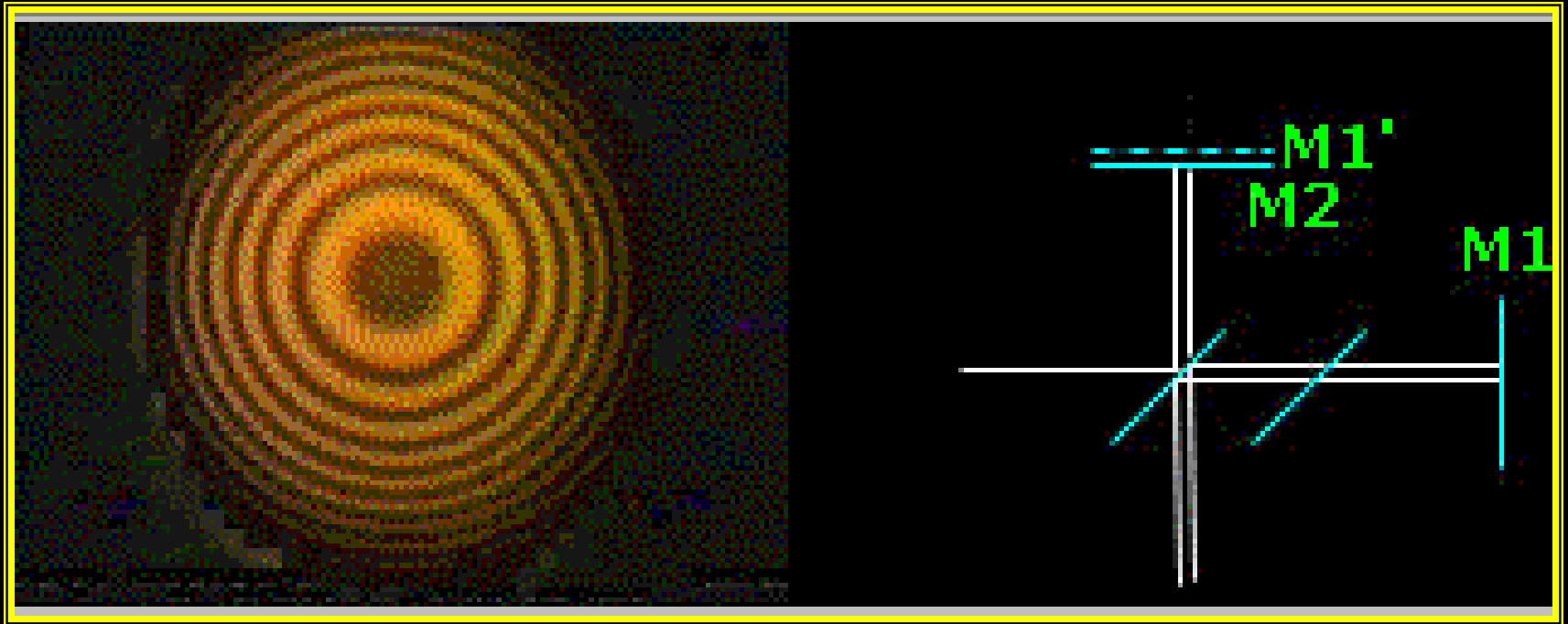


# Michelson interferometer(迈克尔逊干涉仪)



$$\Delta d = N \frac{\lambda}{2}$$

# The interference pattern of Michelson interferometers



**Example** A certain film with index of refraction 1.4 is put into one arm of the interferometer, the fringes move 7 orders, find the thickness of the film (assuming the wavelength  $\lambda = 5893\text{\AA}$ )

**solution**

$$\Delta\delta = 2(n - 1)t = \Delta k\lambda$$

$$t = \frac{\Delta k \cdot \lambda}{2(n - 1)}$$

$$= \frac{7 \times 5893 \times 10^{-10}}{2(1.4 - 1)} = 5.154 \times 10^{-6} \text{ m}$$

