Interference of light waves

Agenda Today

1. Two-slit experiment
2. Film interference
3. Newton’s rings

Conditions for interference

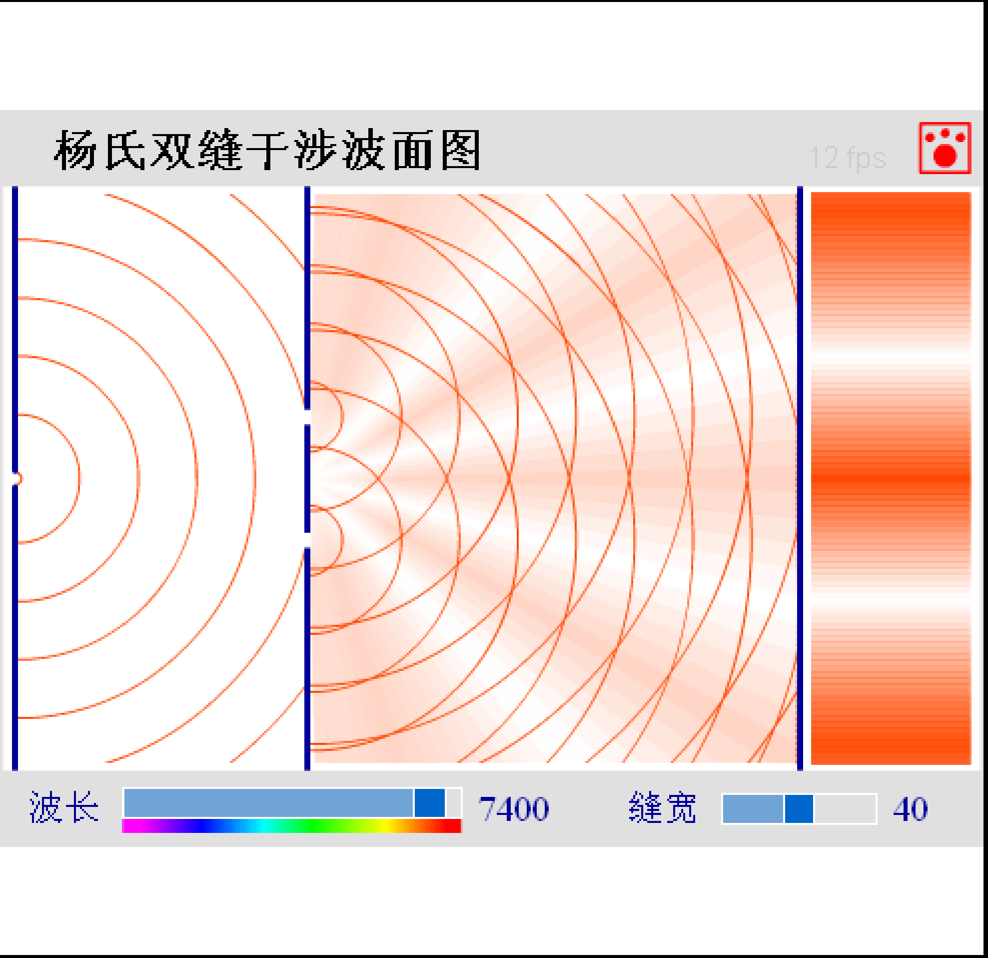
1 the source should be monochromatic(单色)
same frequency

2 the sources must maintain a constant phase difference.

Why is there no interference pattern caused by two bulbs?

Young's double-slit experiment（杨氏双缝实验）

Thomas Young (1773-1829)

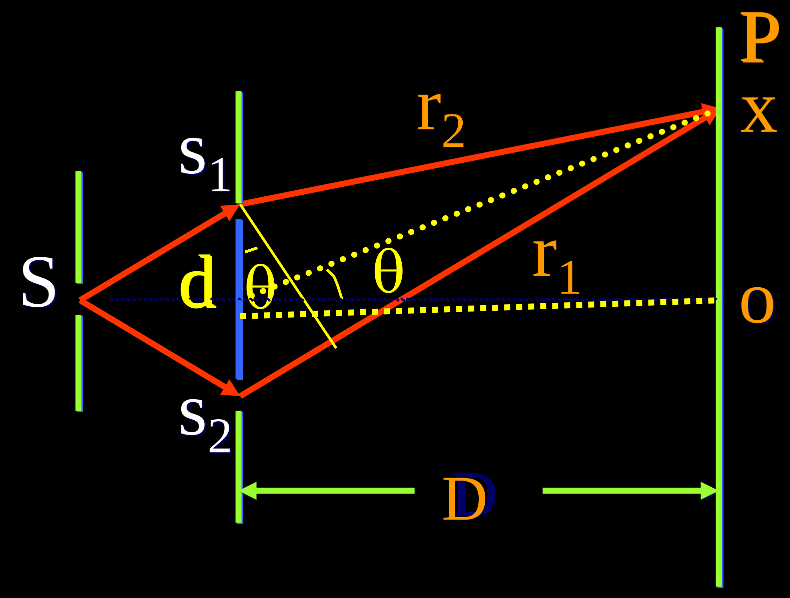


In the case the double slits：

$$\delta = r_2 - r_1 \approx d\sin\theta$$

$$\because D >> a \quad , \theta \; is \; very \; small$$

$$\therefore \sin\theta \approx tg\theta = d\frac{x}{D}$$



$$d\frac{x}{D} = \pm k\lambda \quad (k = 0,1,2,\cdots)$$

Constructive

bright fringes

$$d\frac{x}{D} = \pm(2k-1)\frac{\lambda}{2} \quad (k = 1,2,3,\cdots)$$

Destructive

dark fringes

Intensity distribution of the double-slit interference

$$\text{let } E_1 = E_0 \cos \omega t$$

$$E_2 = E_0 \cos(\omega t + \Delta\Phi)$$

$$E = E_1 + E_2 = 2E_0 \cos\left(\frac{\Delta\Phi}{2}\right)\cos\left(\omega t + \frac{\Delta\Phi}{2}\right)$$

$$\because I \propto E^2$$

$$\therefore I = I_{\max}\cos^2\left(\frac{\Delta\Phi}{2}\right) = I_{\max}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Example: white light passes through two slits 0.50 mm apart and an interference pattern is observed on a screen 2.5 m away. The first-order fringe resembles a rainbow with violet and red light at either end. The violet light falls about 2.0 mm and the red 3.5 mm from the center of the central white fringe. Estimate the wavelengths of the violet and red lights

Solution:

since k=1 , for violet light , we will have:

$$\lambda = \frac{dx}{kD} = 4.0 \times 10^{-7}\, m = 400$$

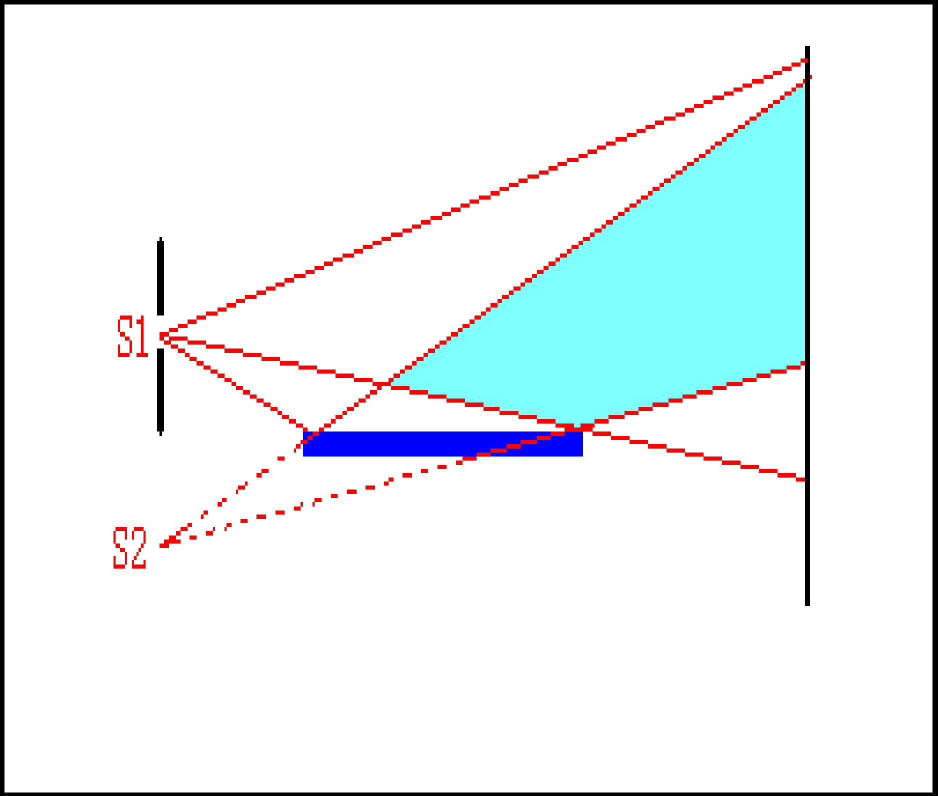
For red light:

$$\lambda = \frac{dx}{kD} = 7.0 \times 10^{-7}\, m = 700$$

Change of phase due to reflection

An electromagnetic wave undergoes a phase change of π upon reflection from a medium that has a higher index of refraction than the one in which the wave is traveling

Lloyd's mirror（洛埃镜）:



Interference in thin films（薄膜干涉）:

Two things must be noted:

1. The phase shift due to reflection.

2. The wavelength of light λ_n in a medium whose refraction index is n is

$$\lambda_n = \frac{\lambda}{n}$$

Where λ is the wavelength of the light in free space

Antireflective coatings(增透膜）

A coating material must have an index of refraction between that of air and the glass to be coated.

$$d_{\min} = \frac{\lambda}{4n}$$

Where λ is the wavelength in vacuum of the rays that we want pass through the coating , n is the refraction index.

The colors of this kind of butterfly do not resemble the same pattern on each side of the wings.

Example:a certain soap film is in the air （n_1=1.33），its thickness is 0.32μm。If a beam of white rays incident on it normally, what is the color of the film when we look downward?

solution: $$2n_1e+\frac{\lambda}{2}=k\lambda \qquad \rightarrow \lambda=\frac{2n_1e}{k-1/2}$$

$$k=1 \quad , \quad \lambda_1=4n_1e=1.70\mu m=17000\dot{A}$$

$$k=2 \quad , \quad \lambda_1=\frac{4}{3}n_1e=0.567\mu m=5670\dot{A}$$

$$k=3 \quad , \quad \lambda_1=\frac{4}{5}n_1e=0.341\mu m=3410\dot{A}$$

λ_2=5670A（黄光）

Quiz: Two flat glass plates of length $L=10$ cm touch at one end but are separated by a wire of diameter $d=0.01$mm at the other end , light shines almost perpendicularly on the glass and is reflected into the eyes, what is the distance x between two bright fringes when $\lambda=400$nm?

Newton's rings

$$2ne + \frac{\lambda}{2} = k\lambda$$

Bright fringes

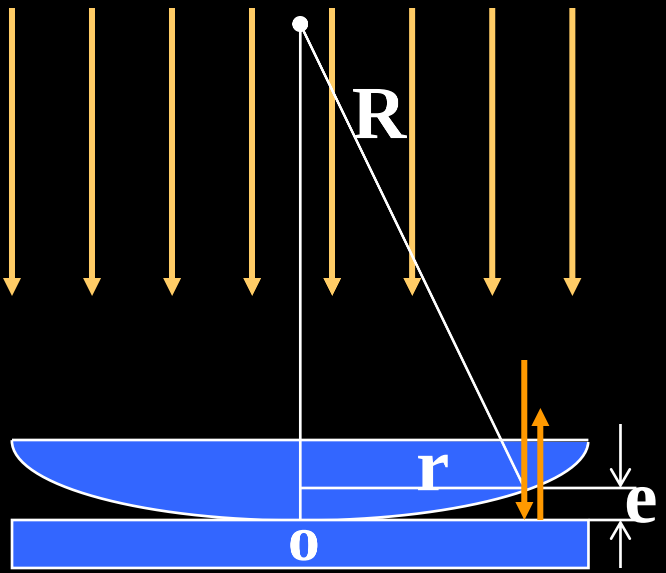
$$2ne + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2}$$

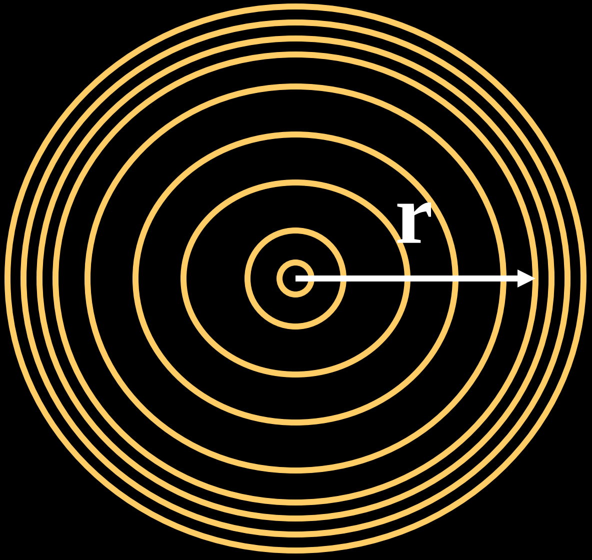
Dark fringes

$$r^2 = R^2 - (R-e)^2 = 2Re - e^2$$

Q $\quad R >> e \rightarrow 2Re >> e^2$

$$e = \frac{r^2}{2R}$$





$$r=\sqrt{\frac{(2k-1)R\lambda}{2n}} \qquad (k=1,2,\cdots)$$

Bright rings

$$r=\sqrt{\frac{kR\lambda}{n}} \qquad (k=0,1,2,\cdots)$$

Dark rings

A certain lamp is used （λ = 5893A）to observe Newton's rings， the radius of dark ring of kth order is r = 4mm， while that of k+5th order is r = 6mm， find the curvature of the lens。

solution:

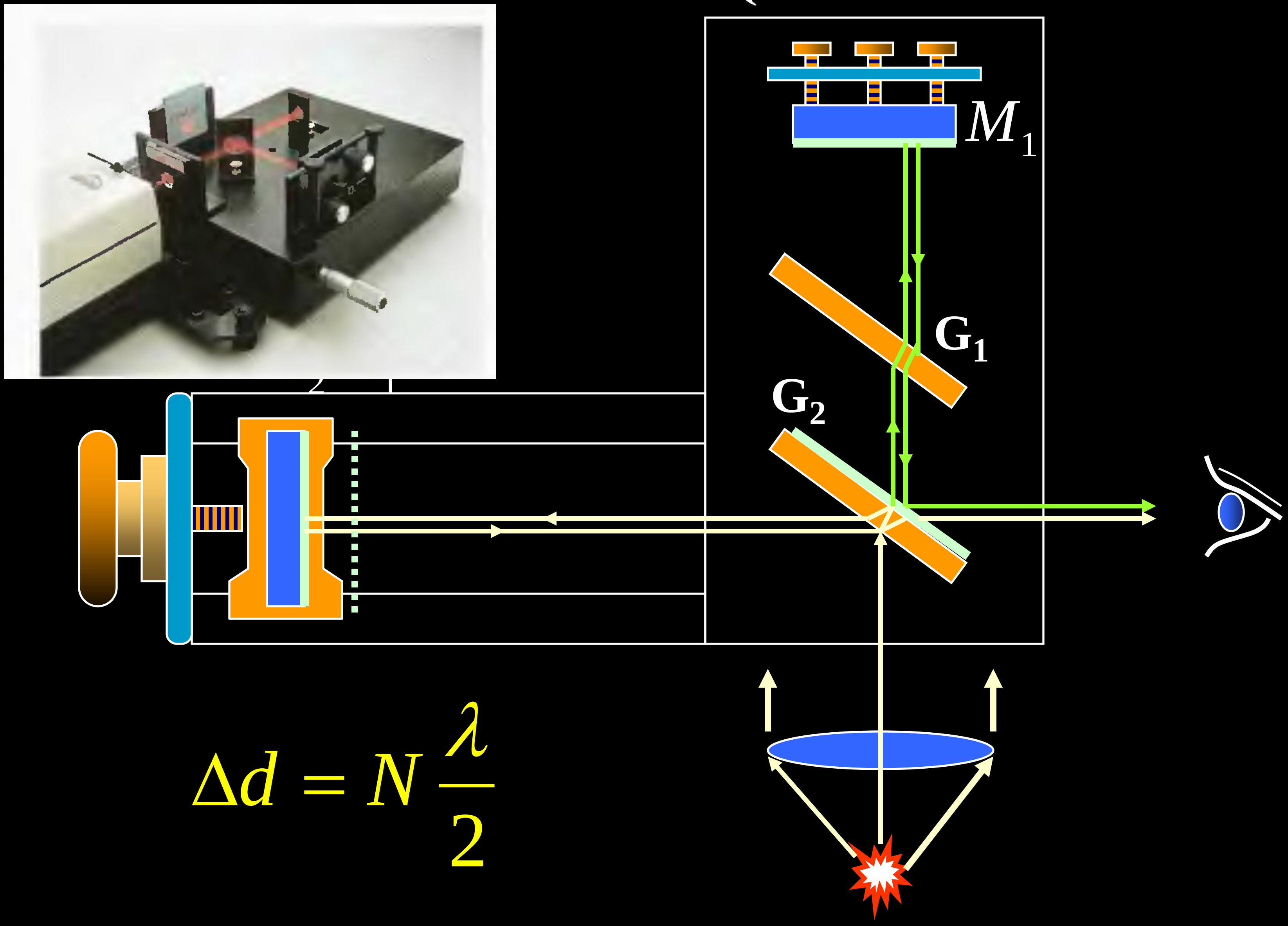
$$r_k = \sqrt{k\lambda R}$$

$$r_{k+5} = \sqrt{(k+5)\lambda R}$$

By solving the equations , we will find

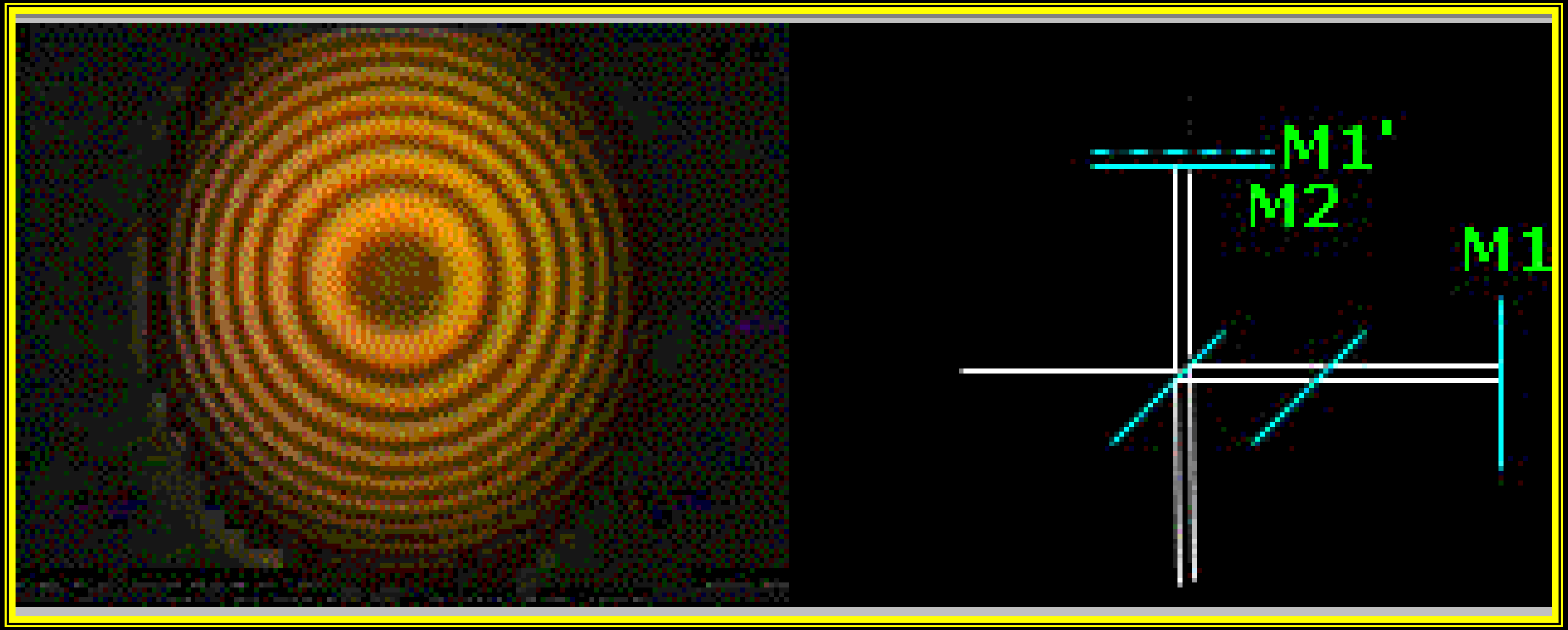
$$k = 4 \qquad R = 6.79\ m$$

Michelson interferometer(迈克尔逊干涉仪）



$$\Delta d = N\frac{\lambda}{2}$$

The interference pattern of Michelson interferometers



Example A certain film with index of refraction 1.4 is put into one arm of the interferometer， the fringes move 7 orders, find the thickness of the film(assuming the wavelength λ = 5893A）

solution

$$\Delta\delta = 2(n-1)t = \Delta k\lambda$$

$$t = \frac{\Delta k \cdot \lambda}{2(n-1)}$$

$$= \frac{7 \times 5893 \times 10^{-10}}{2(1.4-1)} = 5.154 \times 10^{-6} m$$