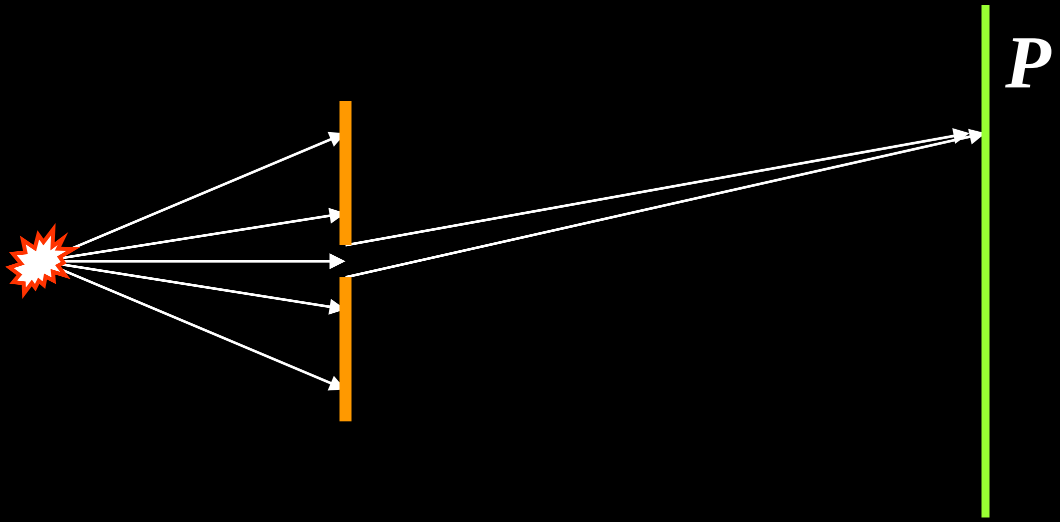
The diffraction of light waves

Agenda today

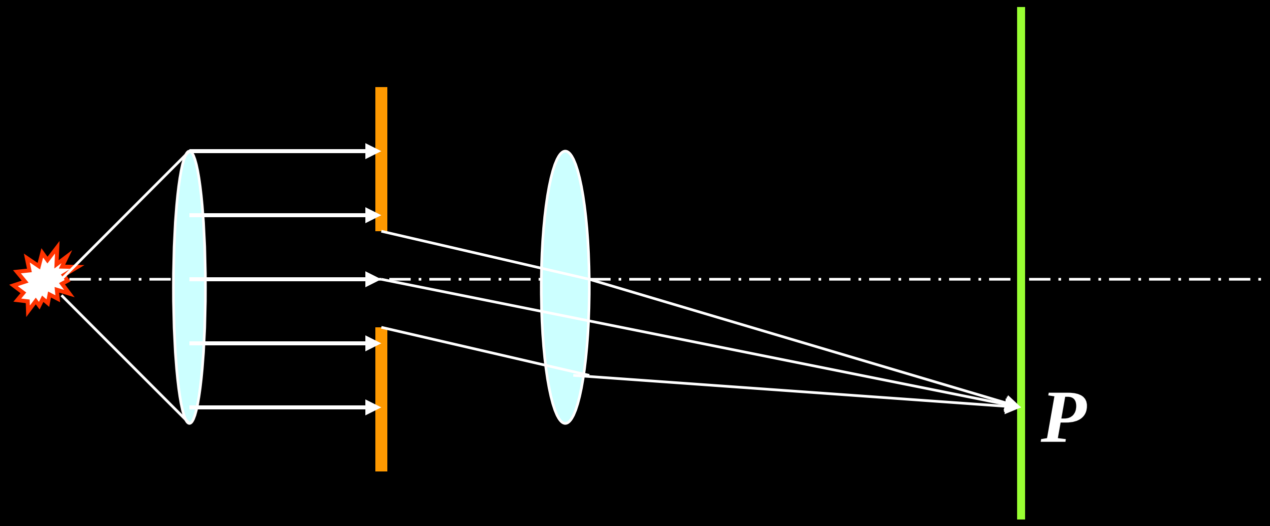
1. Single slit diffraction
2. Diffraction by a circular aperture
3. Optical grating
4. X-ray diffraction32

Diffraction

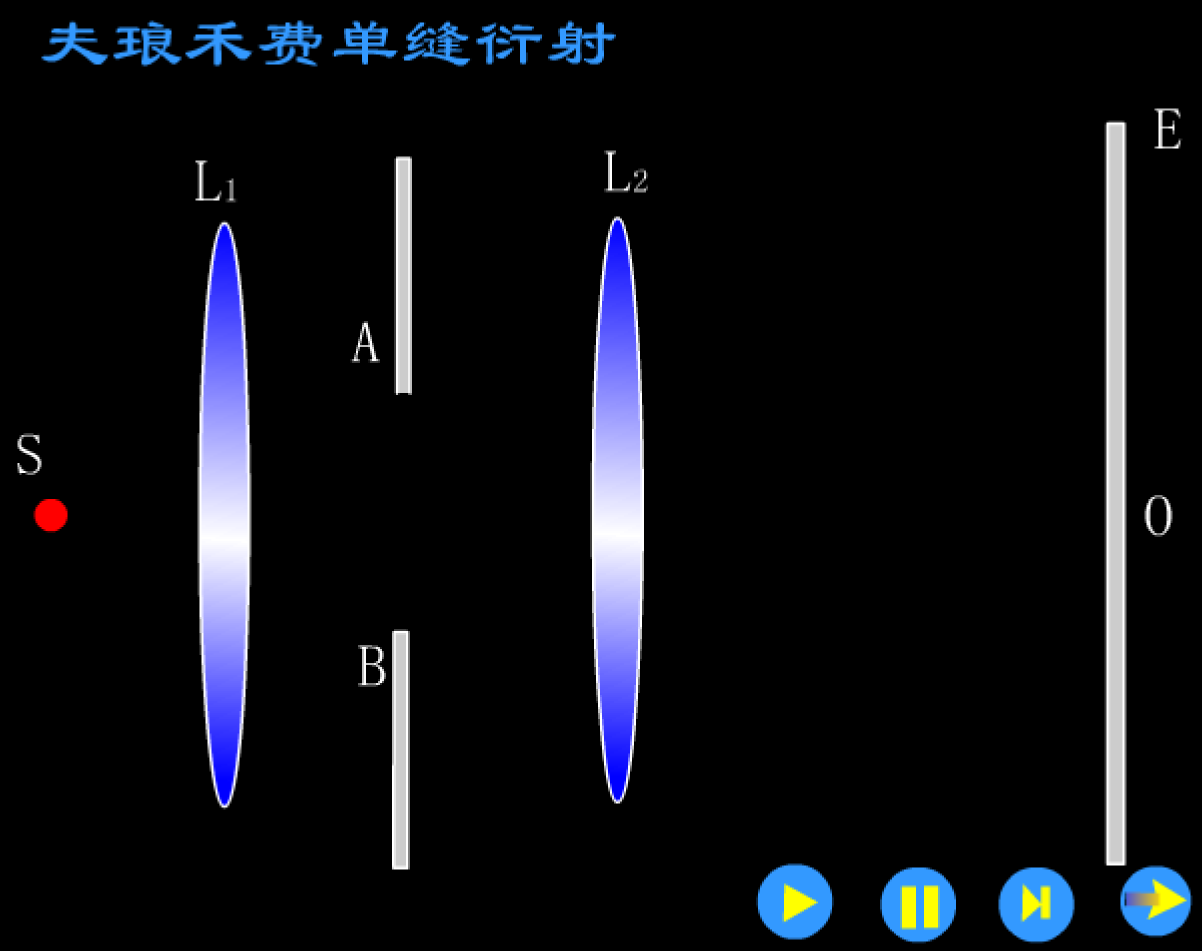
Fresnel diffraction： ***if a source s and the viewing screen are each a finite distance from an intermediate object, the resulting diffraction pattern is called fresnel diffraction(菲涅尔衍射）。***



Fraunhofer diffraction： *if both the source and the screen are far from the intermediate object, the diffraction is called fraunhofer diffraction(夫朗和费衍射）。*



Single slit diffraction(单缝衍射）

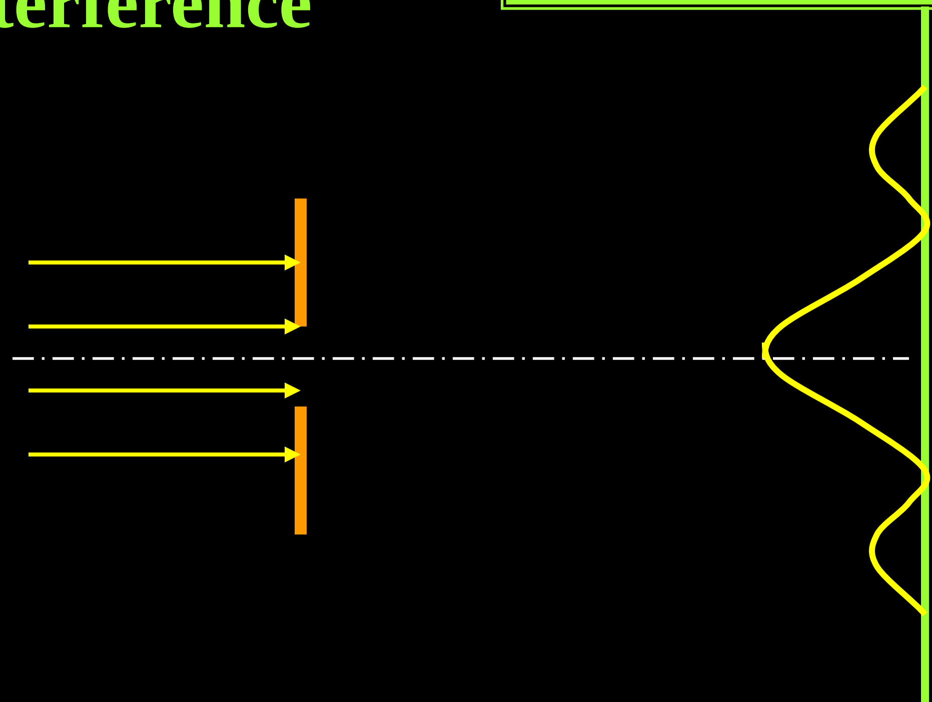


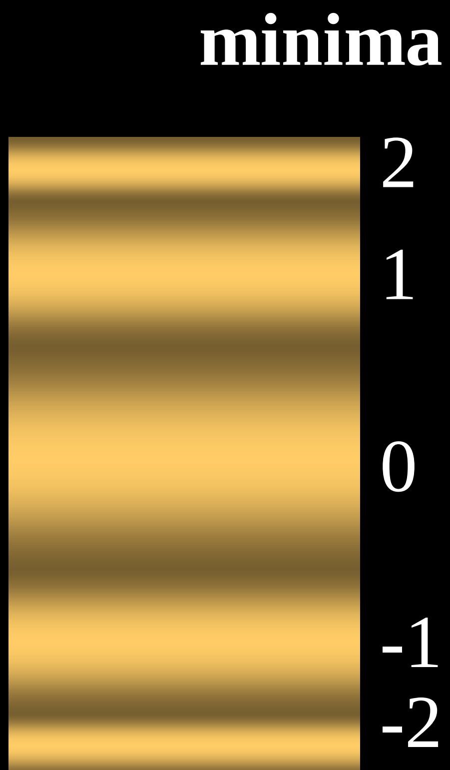
For constructive interference:

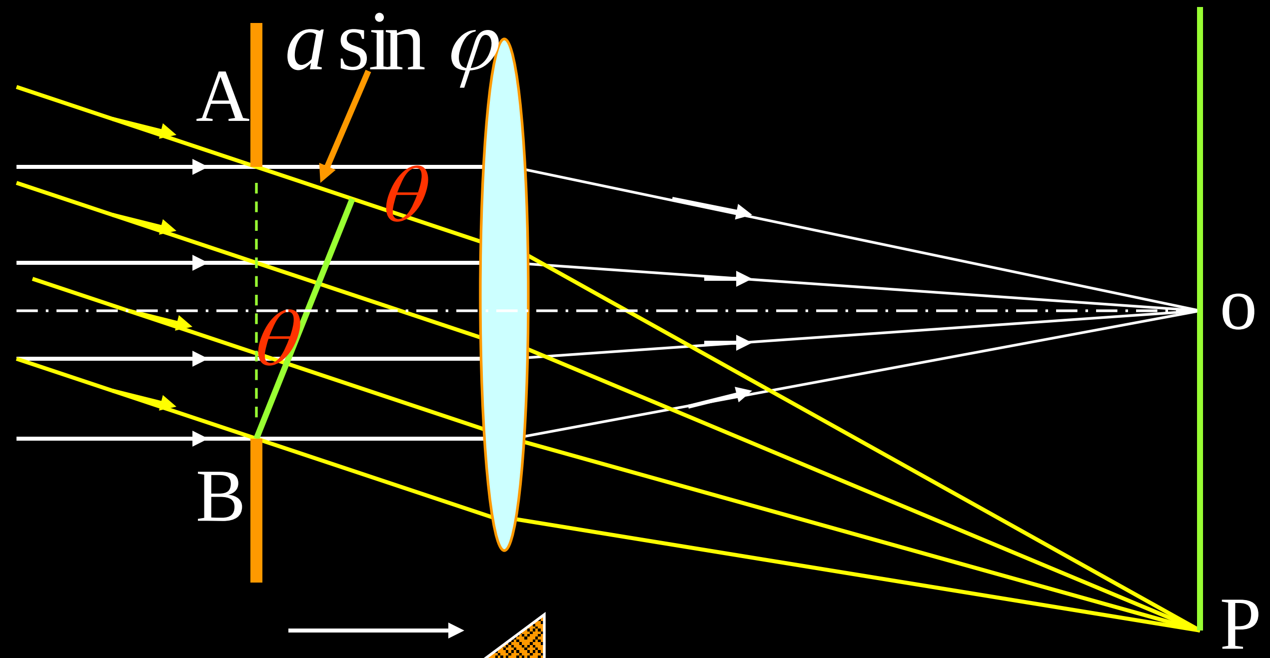
$$a\sin\theta = \pm(2k+1)\frac{\lambda}{2}$$

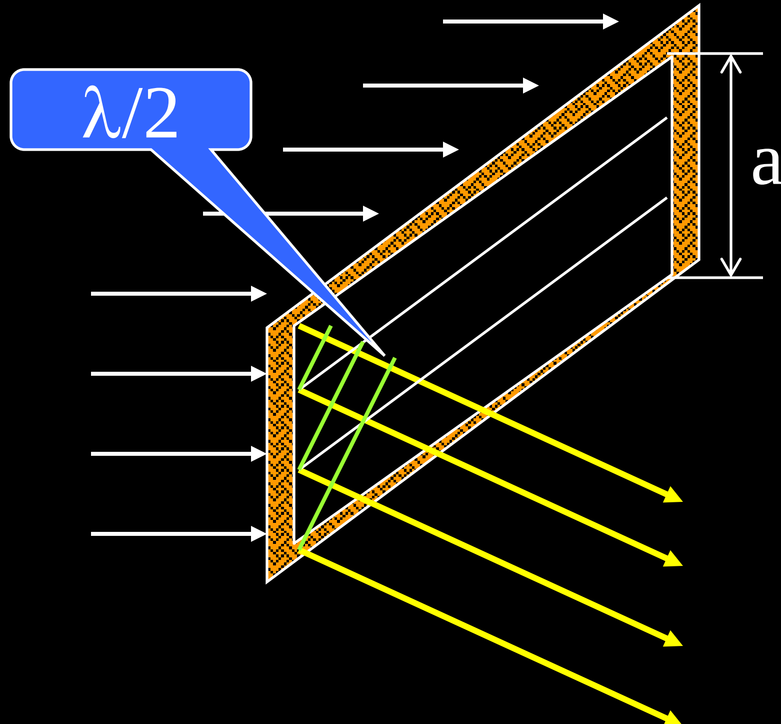
For destructive interference

$$a\sin\theta = \pm k\lambda$$

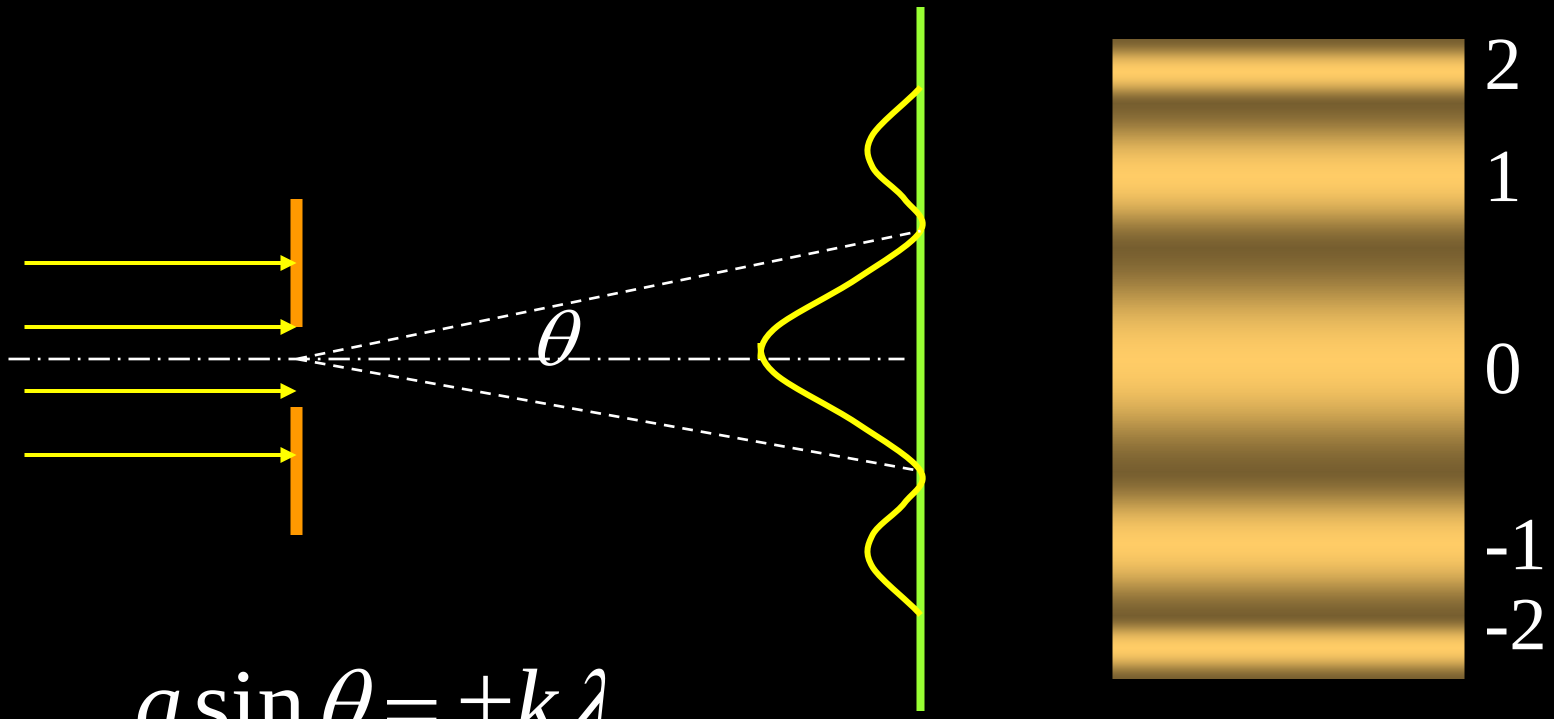








$$a\sin\theta = n\frac{\lambda}{2}$$



$$a\sin\theta = \pm k\lambda$$

the angular width of central maxima：

$$-\lambda < a\sin\theta < \lambda$$

$$\Phi = 2\theta \approx 2\sin\theta = \frac{2\lambda}{a}$$

Diffraction pattern with different slit width:

0.16 mm

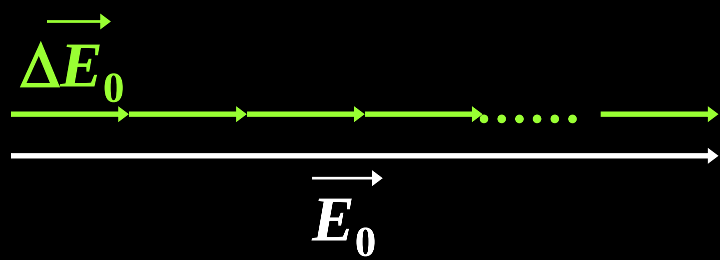
0.08 mm

0.04 mm

0.02 mm

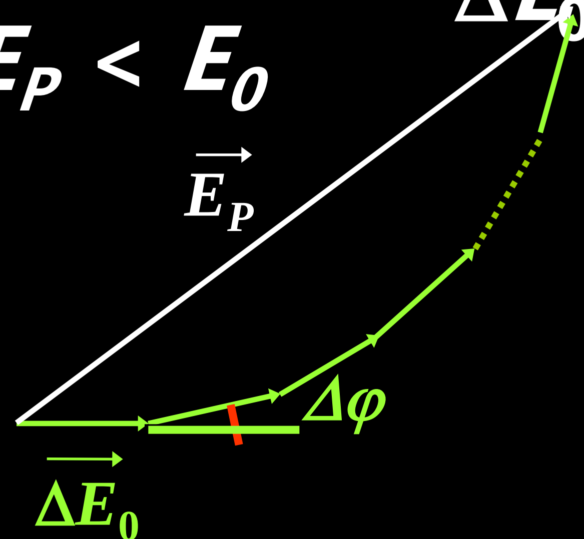
Intensity in single-slit diffraction

For point o $\theta = 0, \ \Delta\varphi = 0$



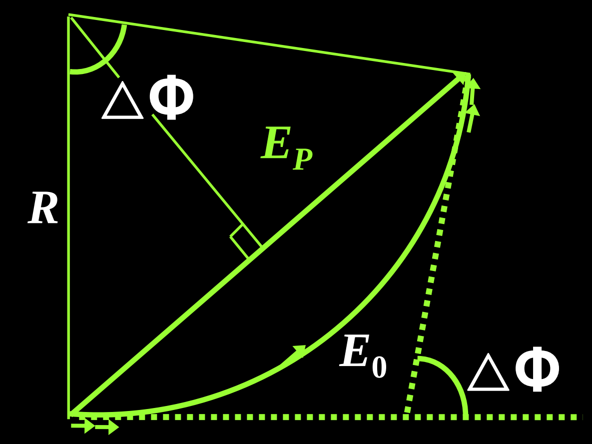
$E_0 = N\Delta E_0$

***For point P*: $E_P < E_0$**



$$\Delta\Phi = \frac{a\sin\theta}{\lambda}2\pi$$

$$E_p = 2R\sin\frac{\Delta\Phi}{2}, \qquad E_0 = R\Delta\Phi$$



$$E_p = 2\frac{E_0}{\Delta\Phi}\sin\frac{\Delta\Phi}{2} = \frac{E_0}{\Delta\Phi/2}\sin\frac{\Delta\Phi}{2}$$

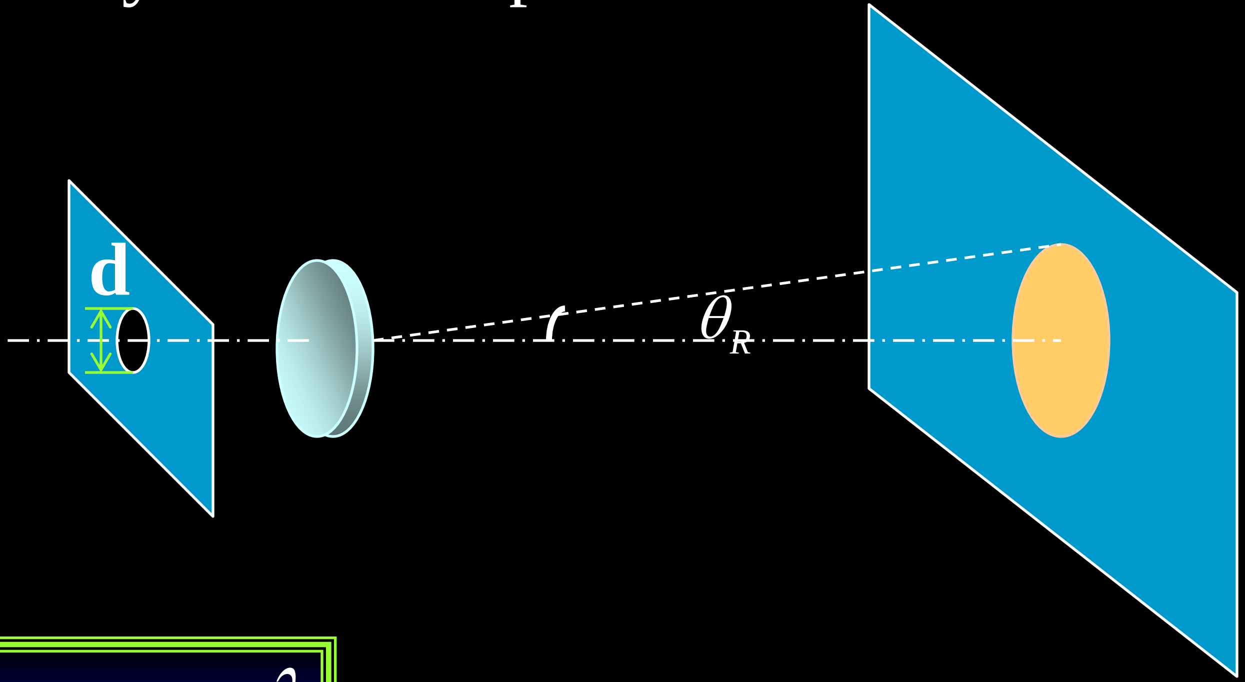
Let $$\alpha = \frac{\Delta\Phi}{2} = \frac{\pi a\sin\theta}{\lambda}$$

$$E_p = E_0\frac{\sin\alpha}{\alpha}$$

$$I \propto E_p^2 \quad , \quad I_0 \propto E_0^2$$

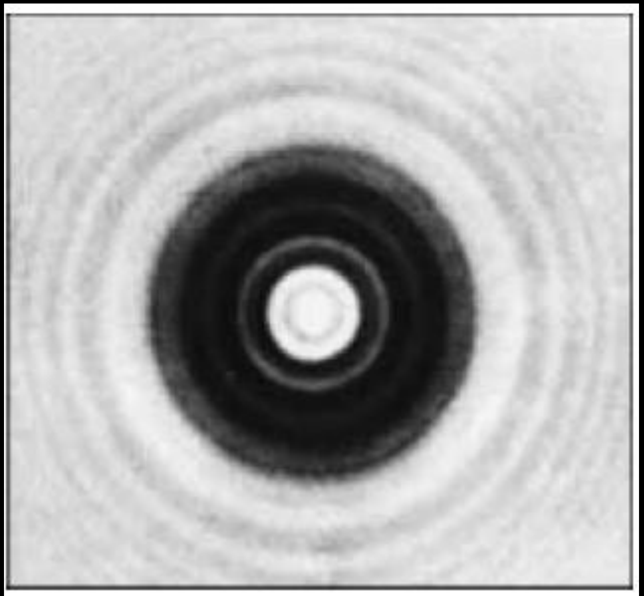
Intensity $$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Diffraction by a circular aperture（圆孔衍射）

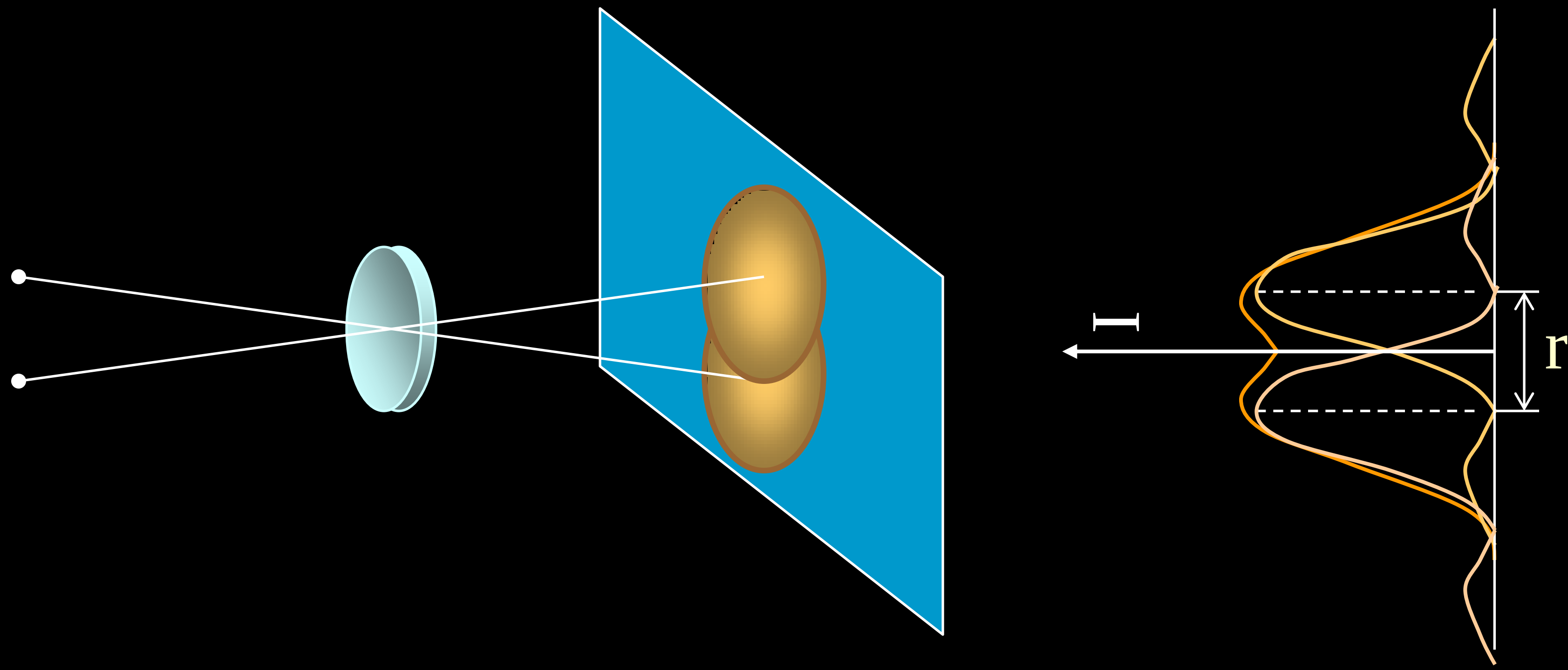


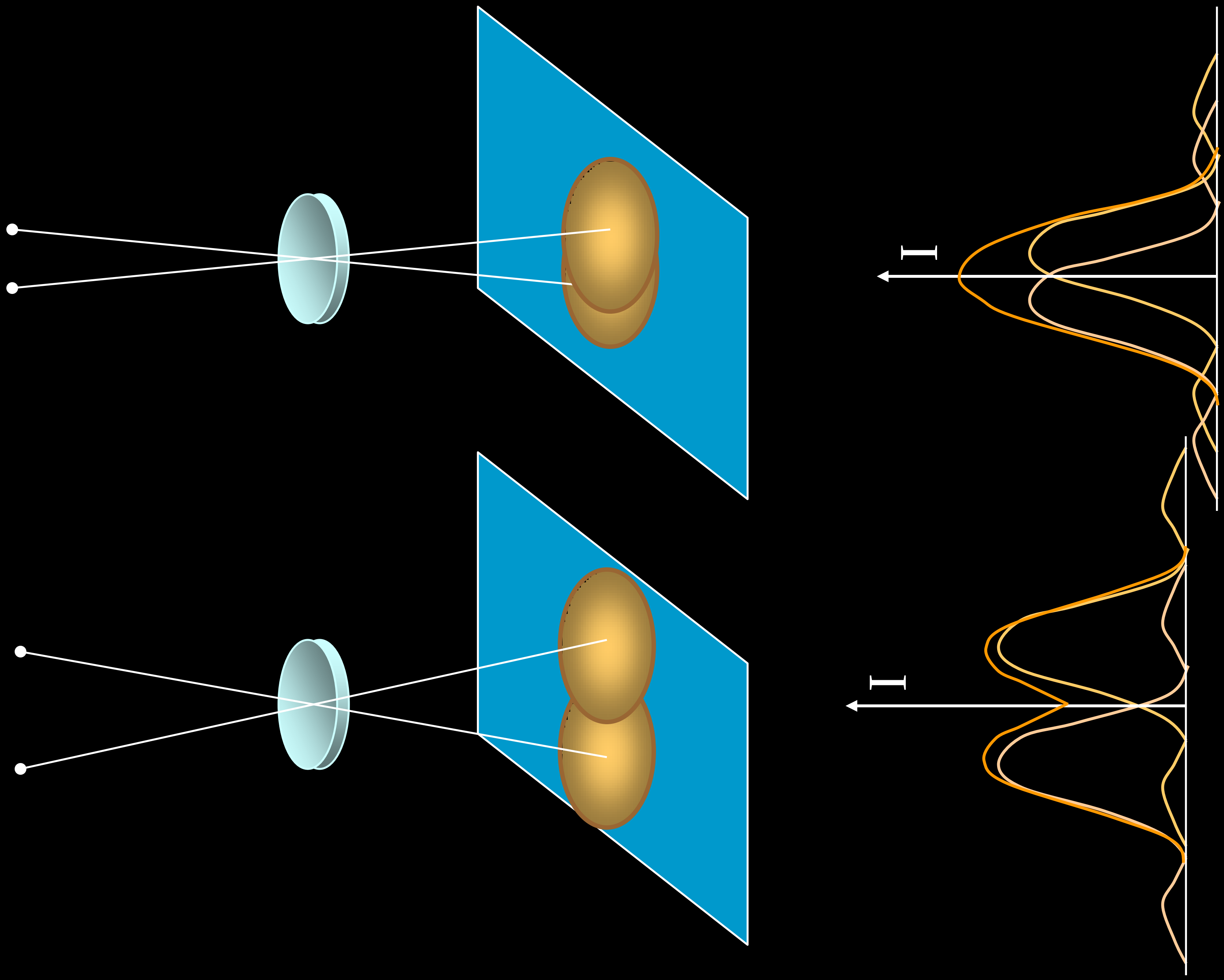
$$\theta_R = 1.22\frac{\lambda}{d}$$

The first minimum (Airy spot):



Rayleigh’s criterion（瑞利判据）: two object are on the verge of resolvability if the central diffraction maximum of one is the first minimum of the other。

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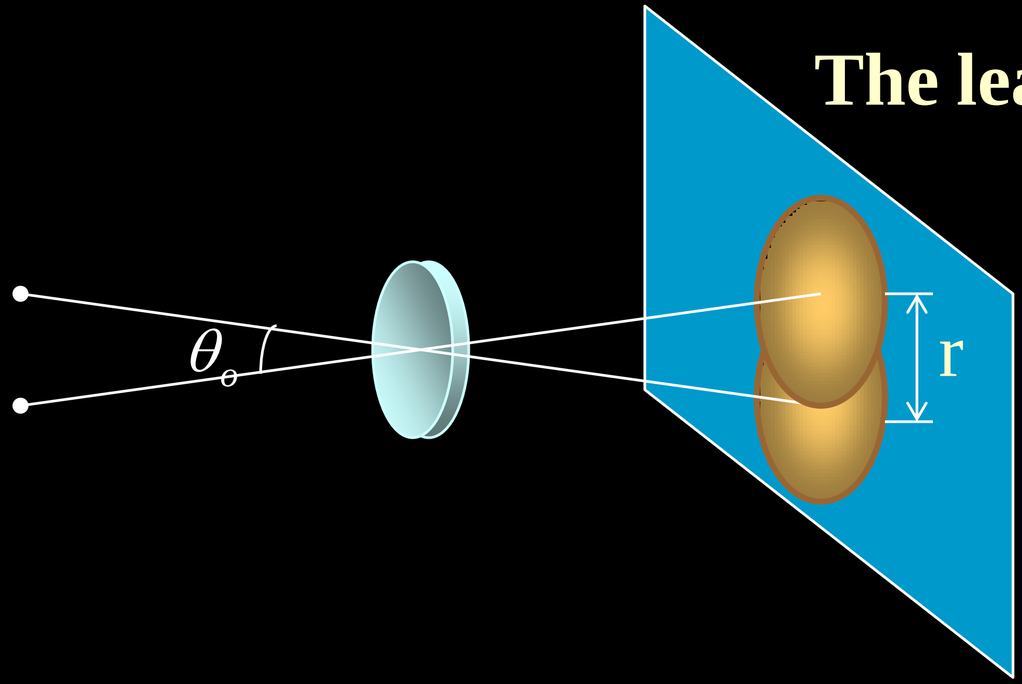


The least angular separation:

$$\theta_0 = 1.22\frac{\lambda}{d}$$

Resolving power（分辨率）:

$$R \equiv \frac{1}{\theta_0} = \frac{d}{1.22\lambda}$$

The resolving power of the optical devices increased with the diameter of its aperture.

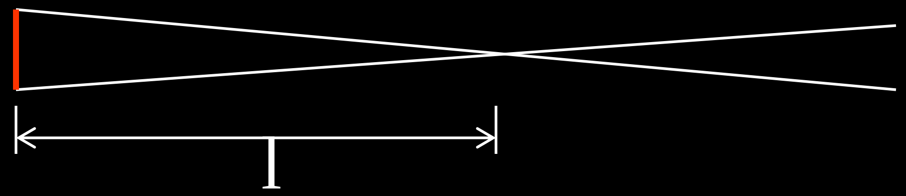
Has the citizen of Liliput a good eyesight ?

Example: the diameter of human's pupil is 3mm，find the least angular separation a man can tell?（λ=5500A）。

solution：

$$\theta_0 = 1.22\frac{\lambda}{d}$$

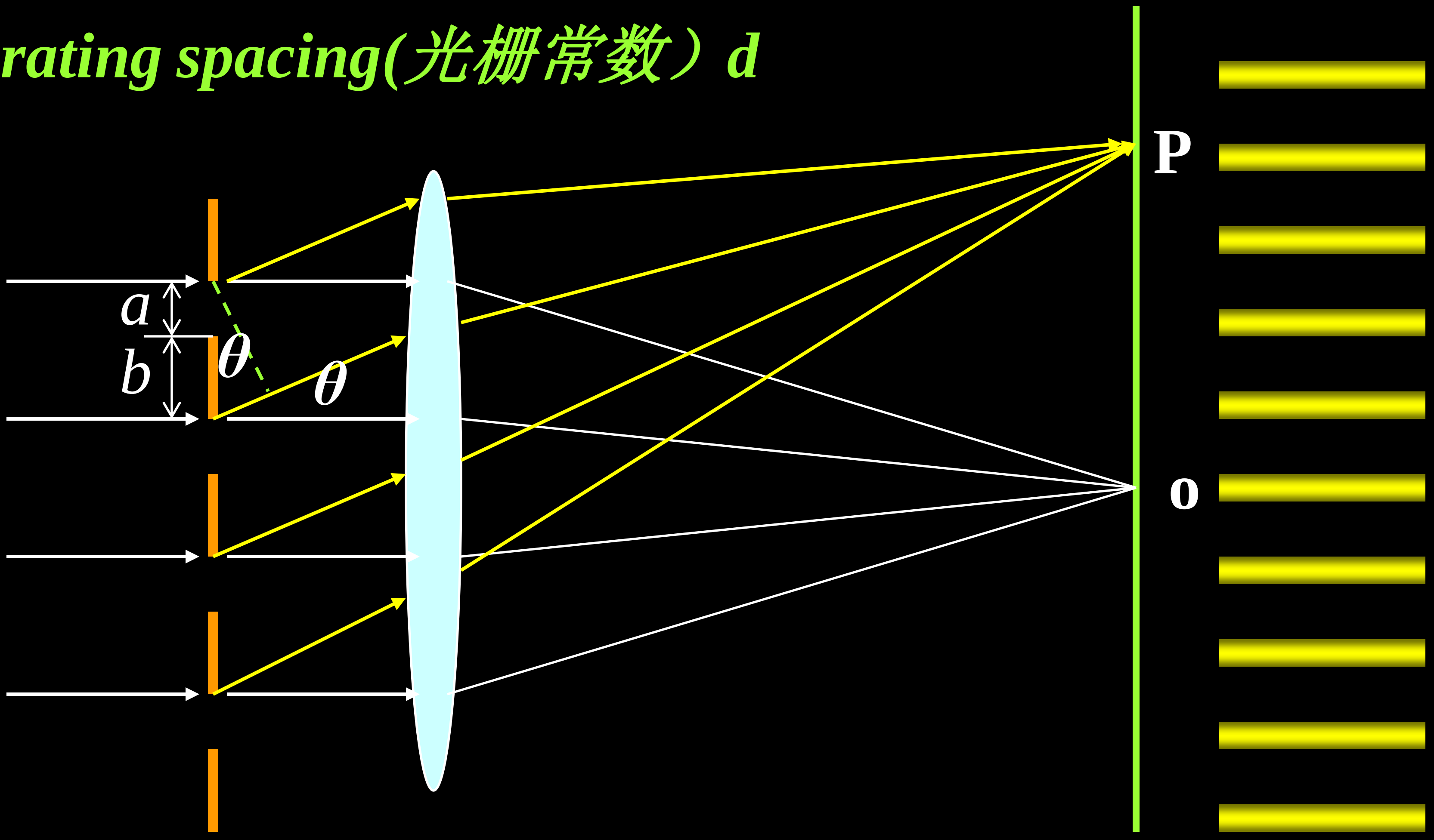
$$= 1.22 \times \frac{5500 \times 10^{-10}}{3 \times 10^{-3}} = 2.2 \times 10^{-4} rad (1')$$



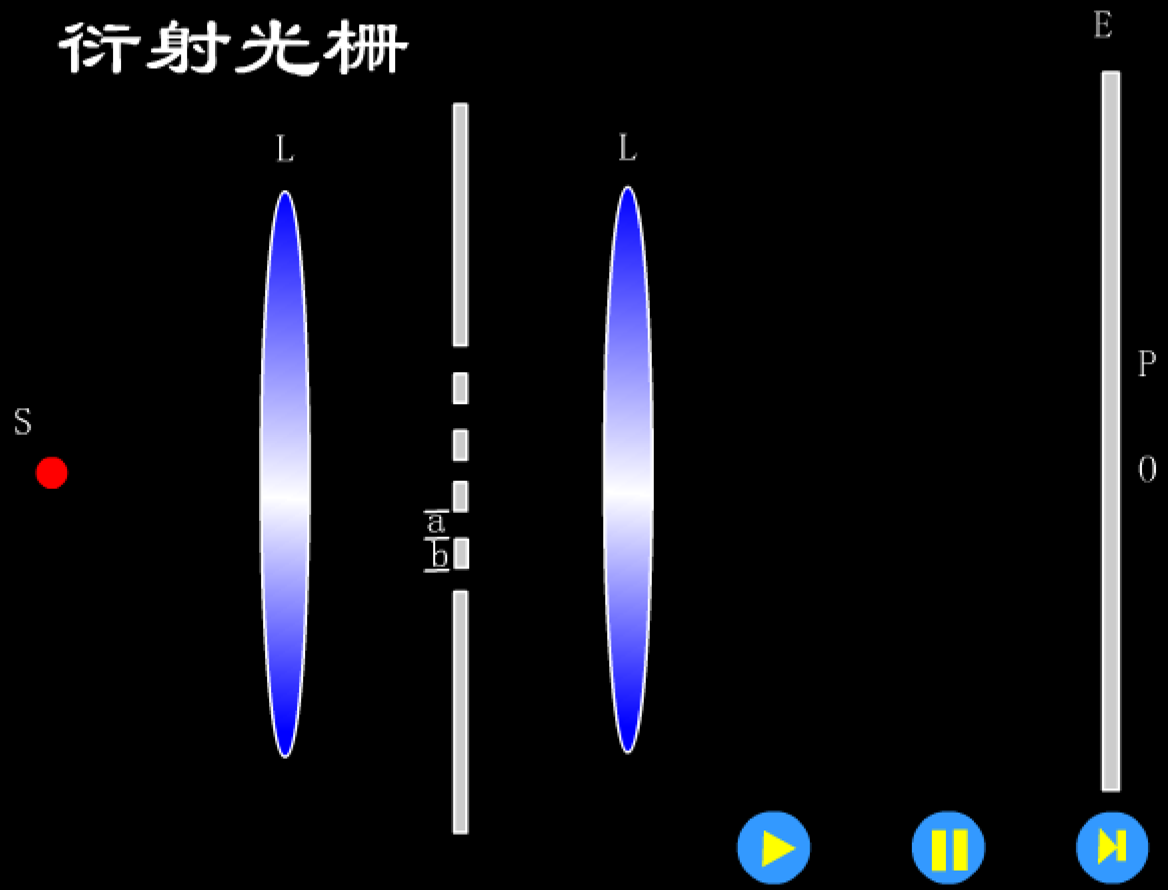
Diffraction grating (衍射光栅）

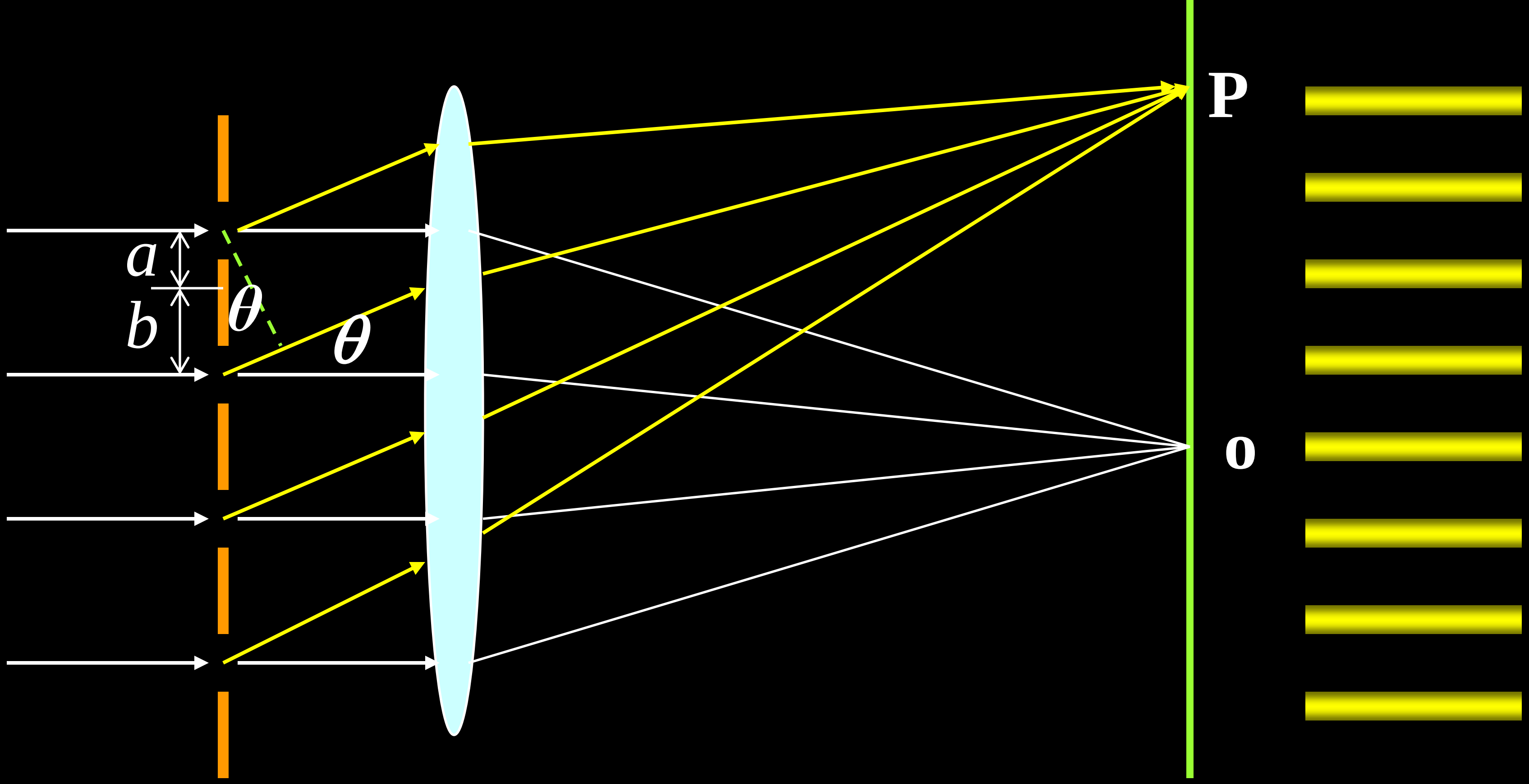
grating： *the screen are regularly divided by slits*

Grating spacing(光栅常数）d



衍射光栅

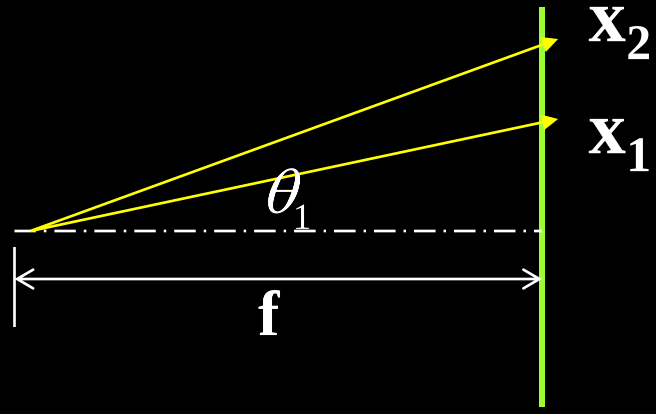




For principle maxima(主极大）：

$$d \sin\theta = \pm m\lambda$$

Example: two monochromic lights (500nm, 520nm)shine on a grating normally, the distance between the slits of the grating is 0.002cm， a set of lens just behind the grating with focus 2m focuses the rays on the screen , find the distance between the third maxima of these two lights.



solution： $$(a+b)\sin\theta = m\lambda$$

$$\sin\theta_1 = \frac{3\lambda_1}{a+b} \qquad \sin\theta_2 = \frac{3\lambda_2}{a+b}$$

$$x_1 = f\cdot tg\theta_1 \qquad x_2 = f\cdot tg\theta_2$$

$$\sin\theta \approx tg\theta$$

$$\Delta x = f(tg\theta_2 - tg\theta_1) = f\left(\frac{3\lambda_2}{a+b} - \frac{3\lambda_1}{a+b}\right) = 0.006\ m$$

The height of the principal maxima increases as N^2,but the width of it decreases as 1/N.

$$E_{max} = NE_0$$

$$I_{max} = N^2 I_0$$

$$I_{max} \Delta\theta \propto N I_0$$

$$\Delta\theta_{hw} = \frac{\lambda}{N d \cos\theta} \propto \frac{1}{N}$$

Dispersion and Resolving Power

Dispersion:

$$D = \frac{\Delta\theta}{\Delta\lambda} \qquad D = \frac{m}{d\cos\theta}$$

Resolving Power:

$$R = \frac{\lambda_{av}}{\Delta\lambda} \qquad R = Nm$$

Order missing:

let: $$\frac{(a+b)}{a}=\frac{m}{n}$$

At the mth order maxima of the grating: $$(a+b)\sin\theta=\pm m\lambda$$

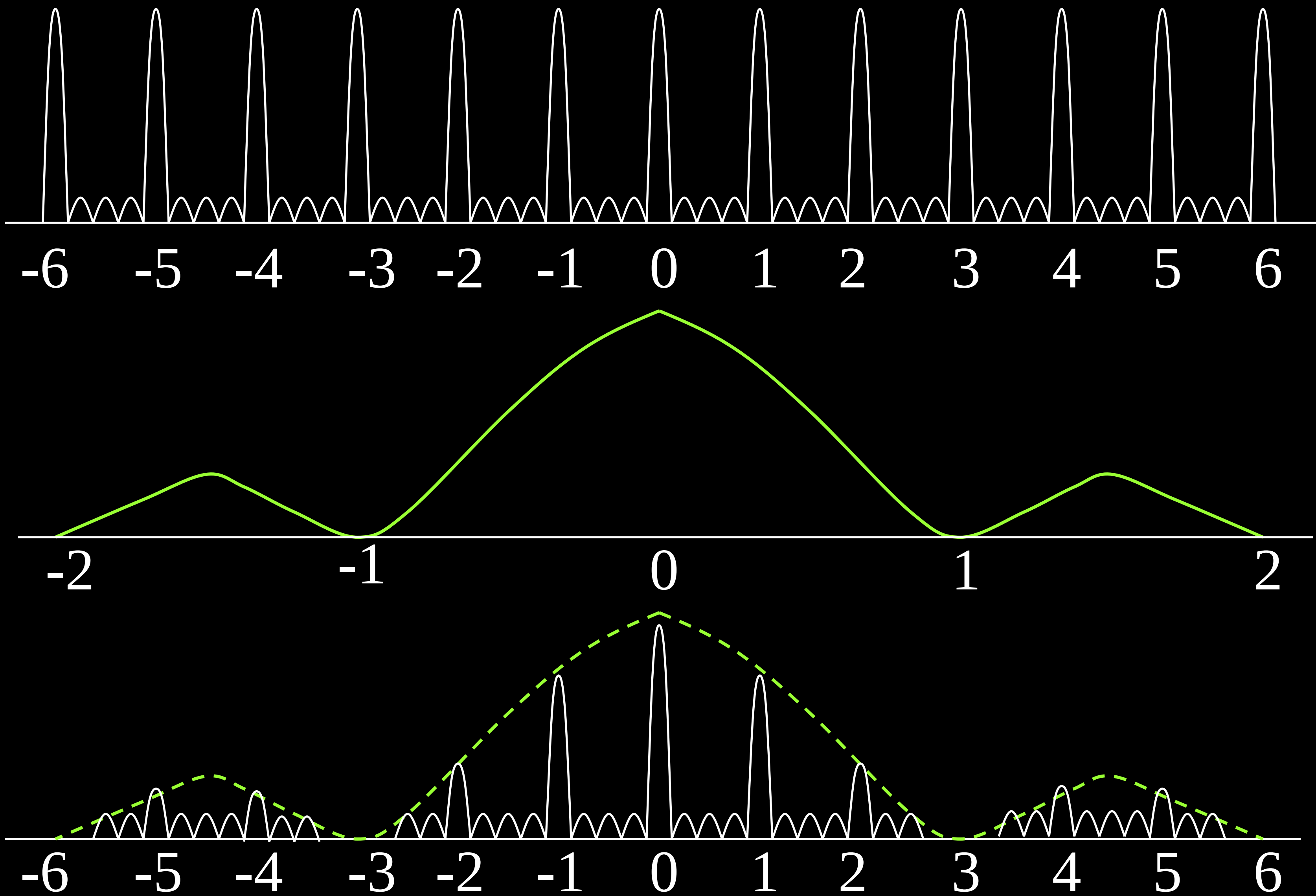
The silt diffraction has its nth order of minima $$\rightarrow a\sin\theta=\pm n\lambda$$

$$k=m,2m,3m,\cdots\cdots$$

$$k'=n\ ,2\,n,\ 3\,n\,,\cdots\cdots$$

Missing order

The effect of slit width on grating patterns



Example: a monochromic light **(600nm)** shines on the grating， the 2nd maxima and 3rd maxima appear at sinθ=0.20,sinθ=0.30， the 4th order is missing。Find a) the grating spacing b) the minimum width of the slit c) all the orders of maxima that could be seen。

solution:

$$(a+b)\sin\theta = \pm m\lambda$$

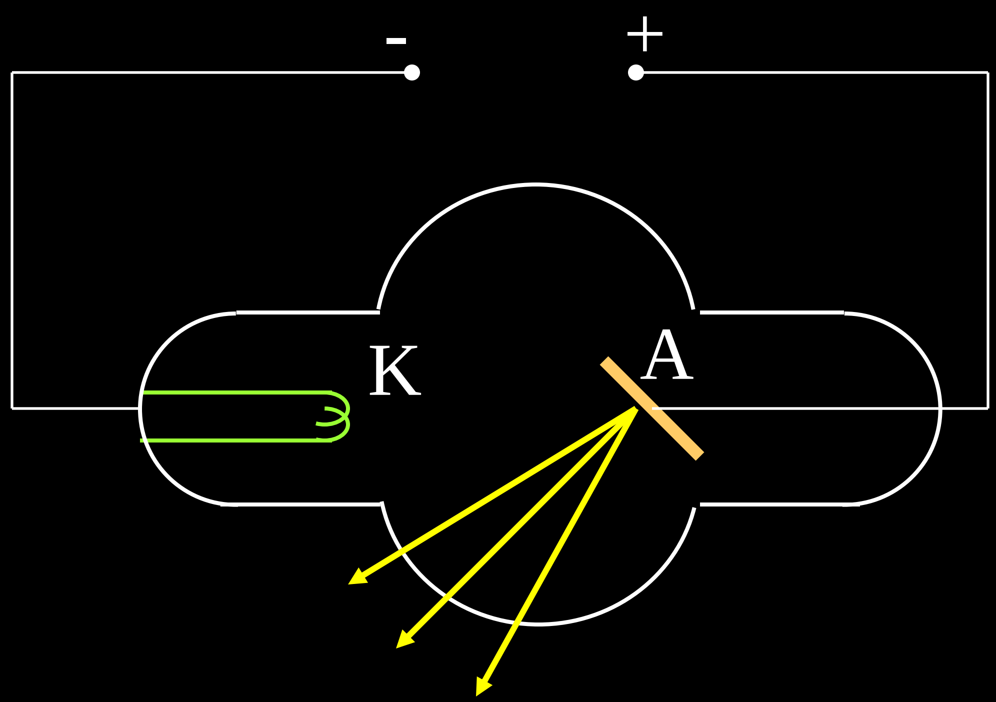
$$(a+b) = \frac{k\lambda}{\sin\theta} = \frac{2\times 6000\times 10^{-10}}{0.2} = 6\times 10^{-6}m$$

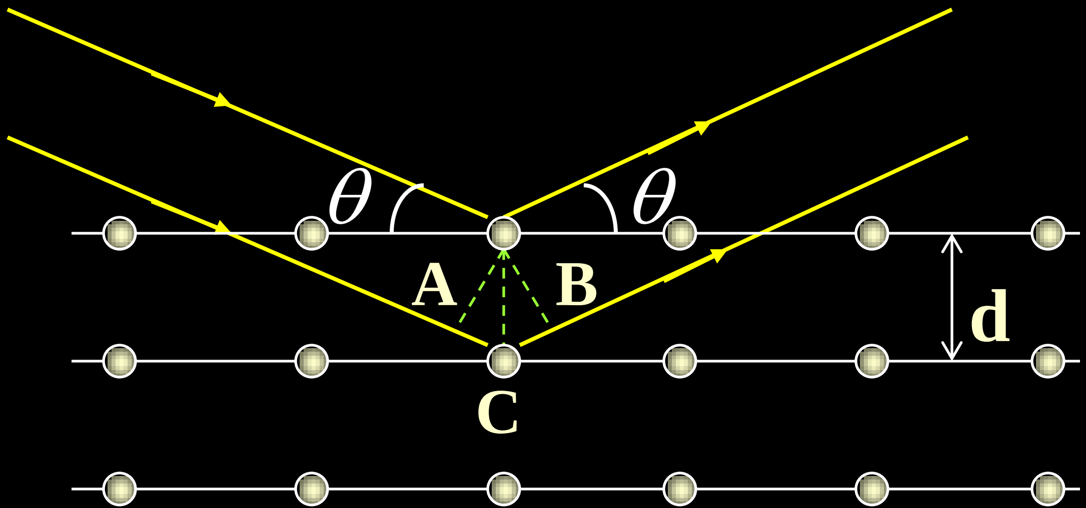
$$a = \frac{(a+b)}{4} = 1.5\times 10^{-6}m \quad k = \frac{a+b}{\lambda}\sin\theta = \frac{6\times 10^{-6}\times 1}{6\times 10^{-7}} = 10$$

$$k = 0,\pm 1,\pm 2,\pm 3,\pm 5,\pm 6,\pm 7,\pm 9$$

Diffraction of x-rays

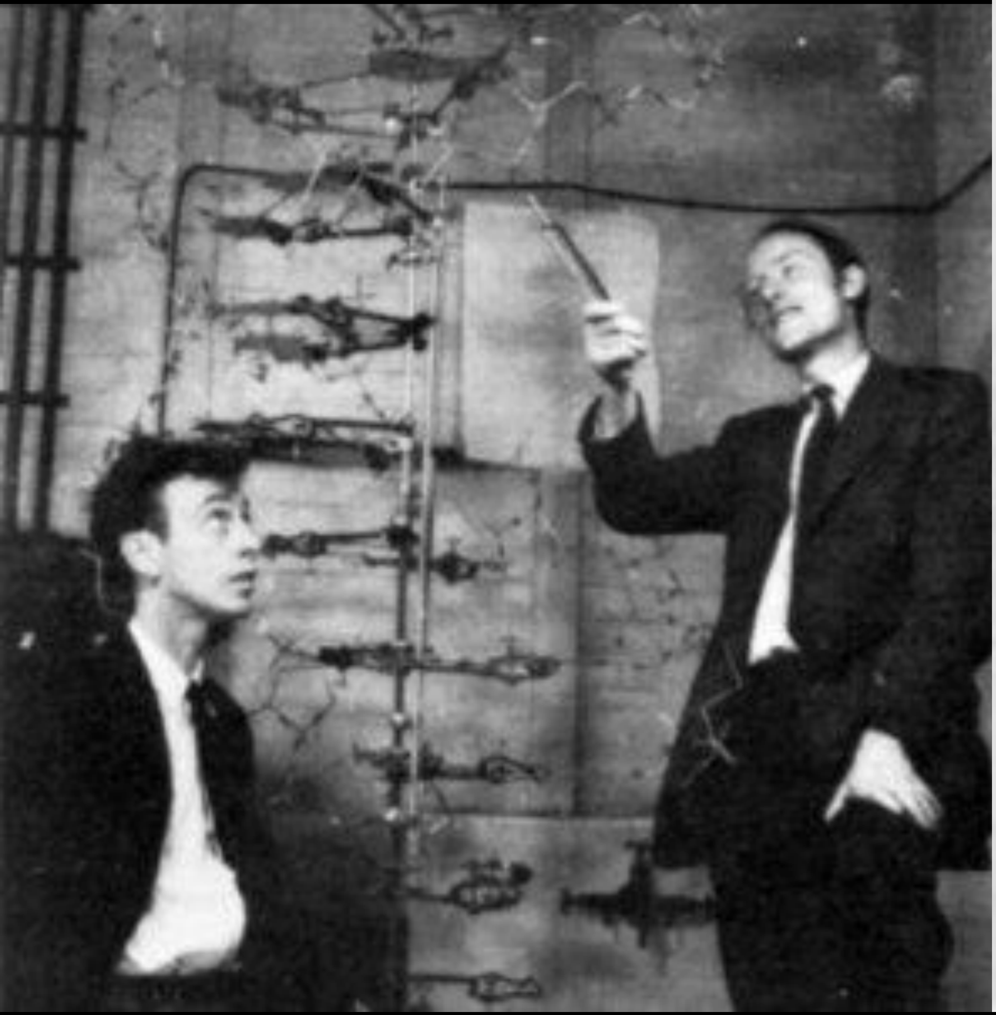
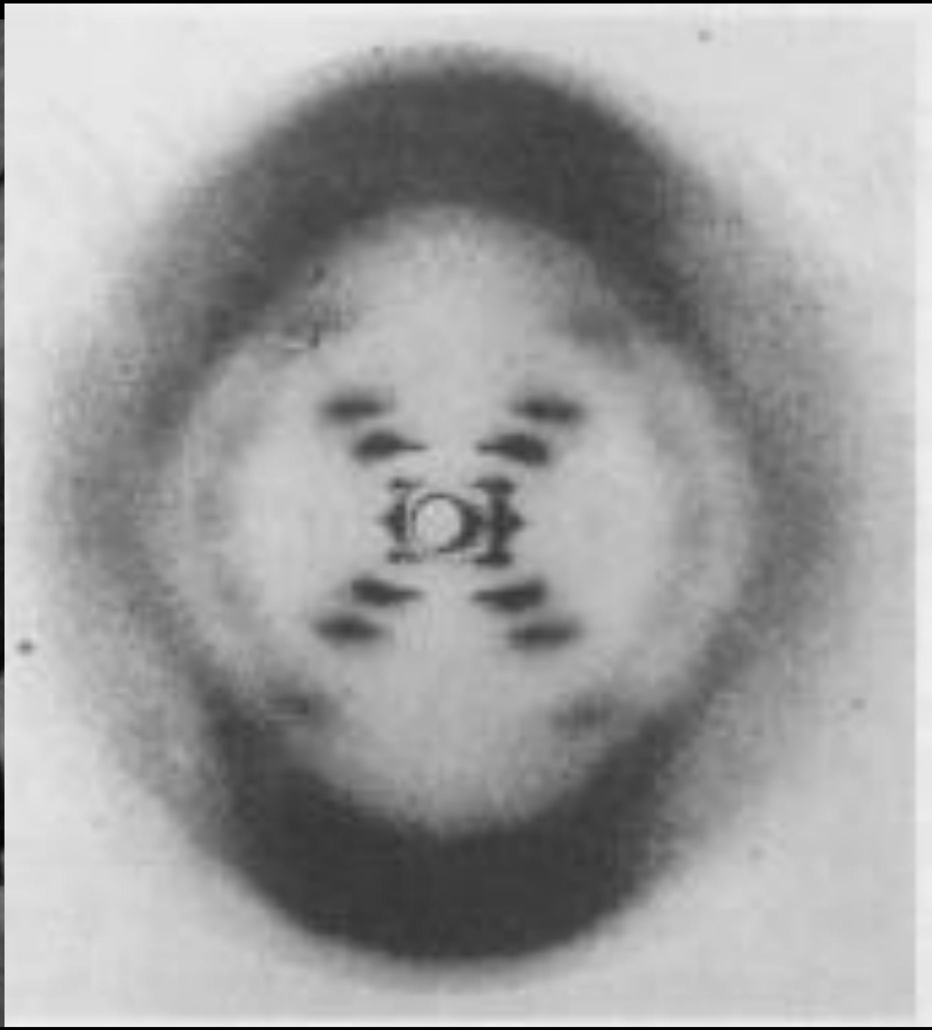
X rays are electromagnetic radiation with wavelength about 0.001~0.01(nm) , which is about the diameter of atoms





Bragg's equation:(布喇格定律)

$$2d\sin\theta = m\lambda \qquad k = 1,2,3,\cdots\cdots$$

Decipher the Structure of DNA