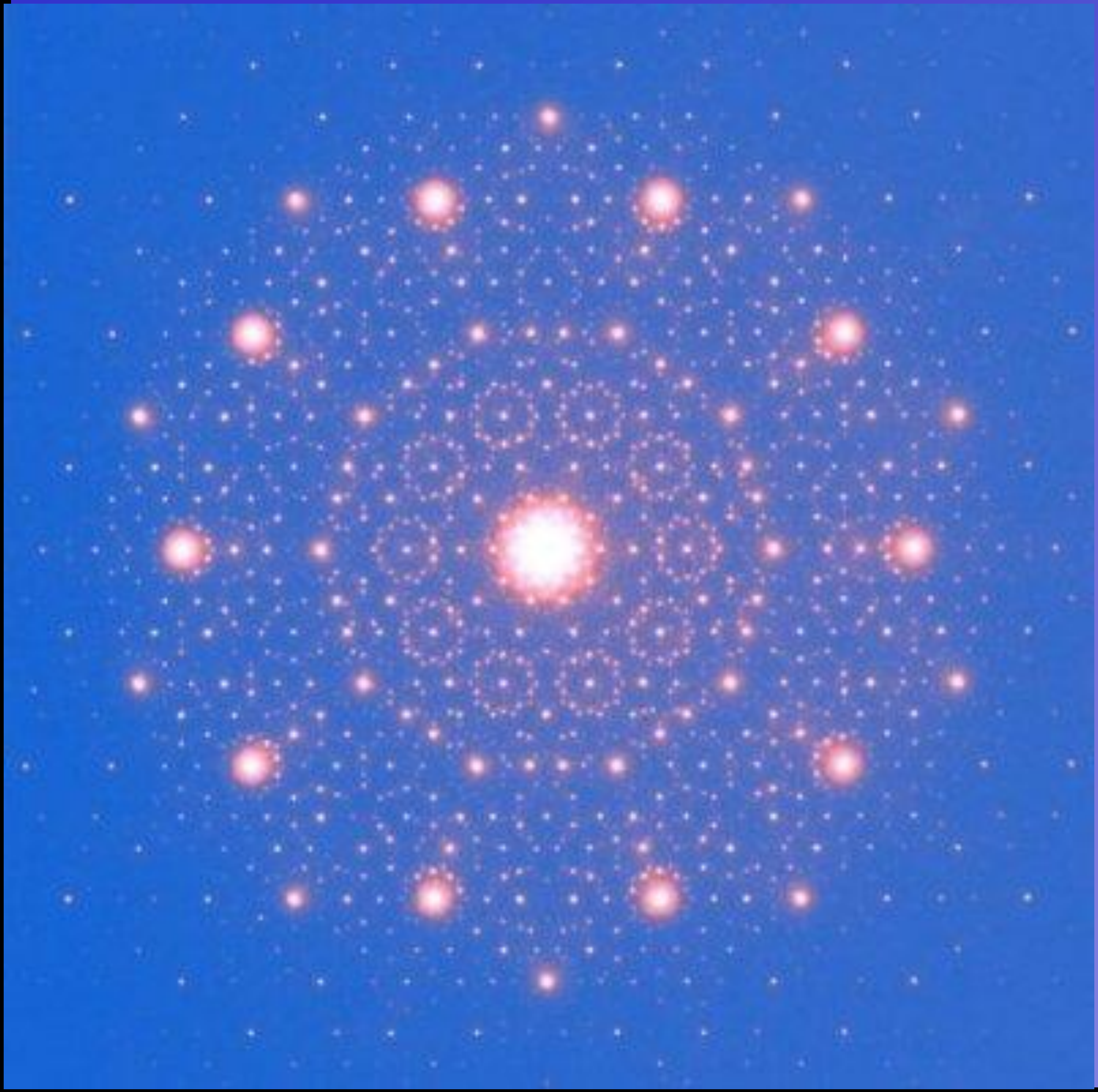


Duality

Agenda today

1. The duality of particle
2. The experiment evidence for wave nature of particles
3. Heisenberg’s uncertainty relations

Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective positions of the things which compose it...nothing would be uncertain, and the future as the past would be laid out before its eyes.

Pierre Simon de Laplace, 1776

Matter waves（物质波）

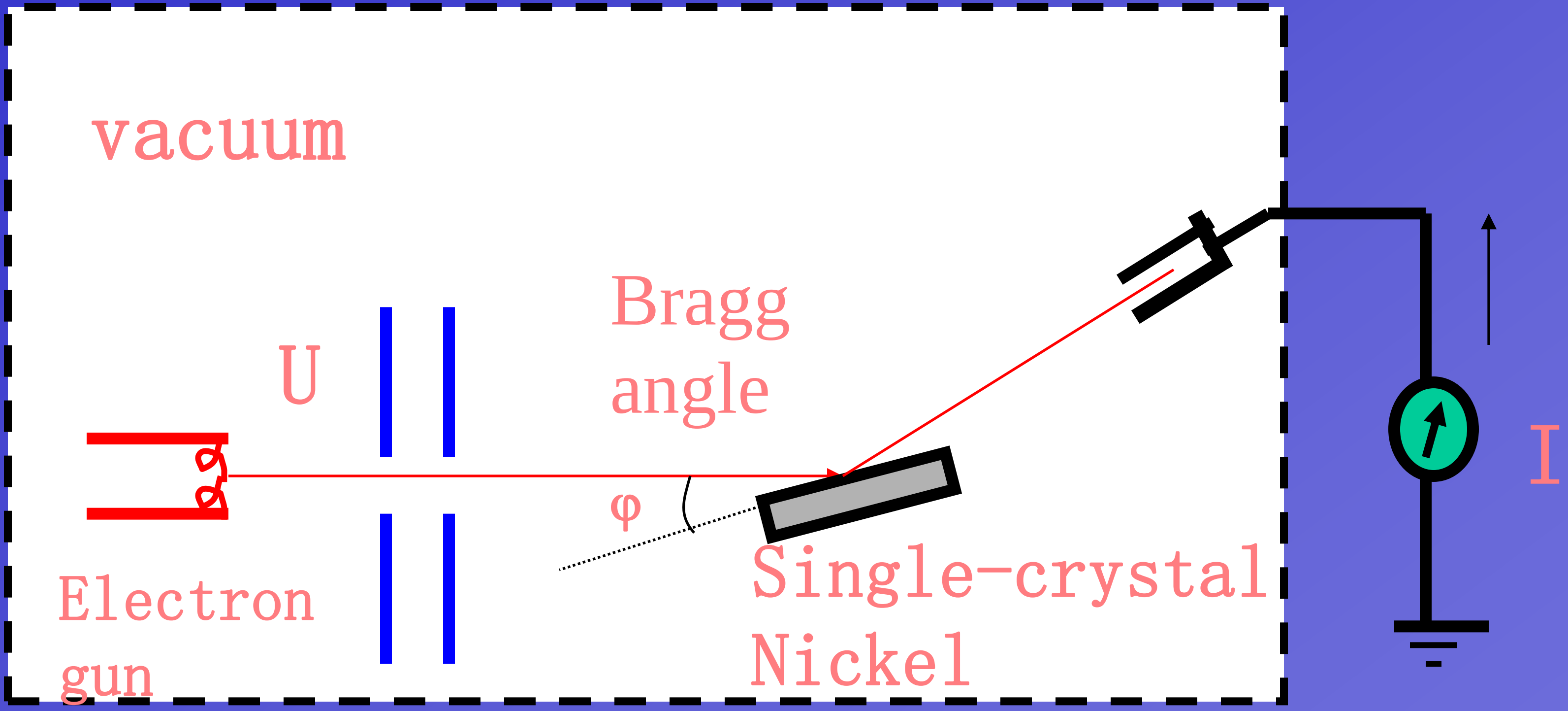
De Borglie suggested that particles as electrons also have wavelike property.

$$\varepsilon = hf$$

$$p = \frac{h}{\lambda}$$

Where λ is called de borglie wavelength

Davisson-Germer experiment(1927)（戴维逊革末实验）



$$\lambda = \frac{h}{p} = \frac{h}{m_0 v}$$

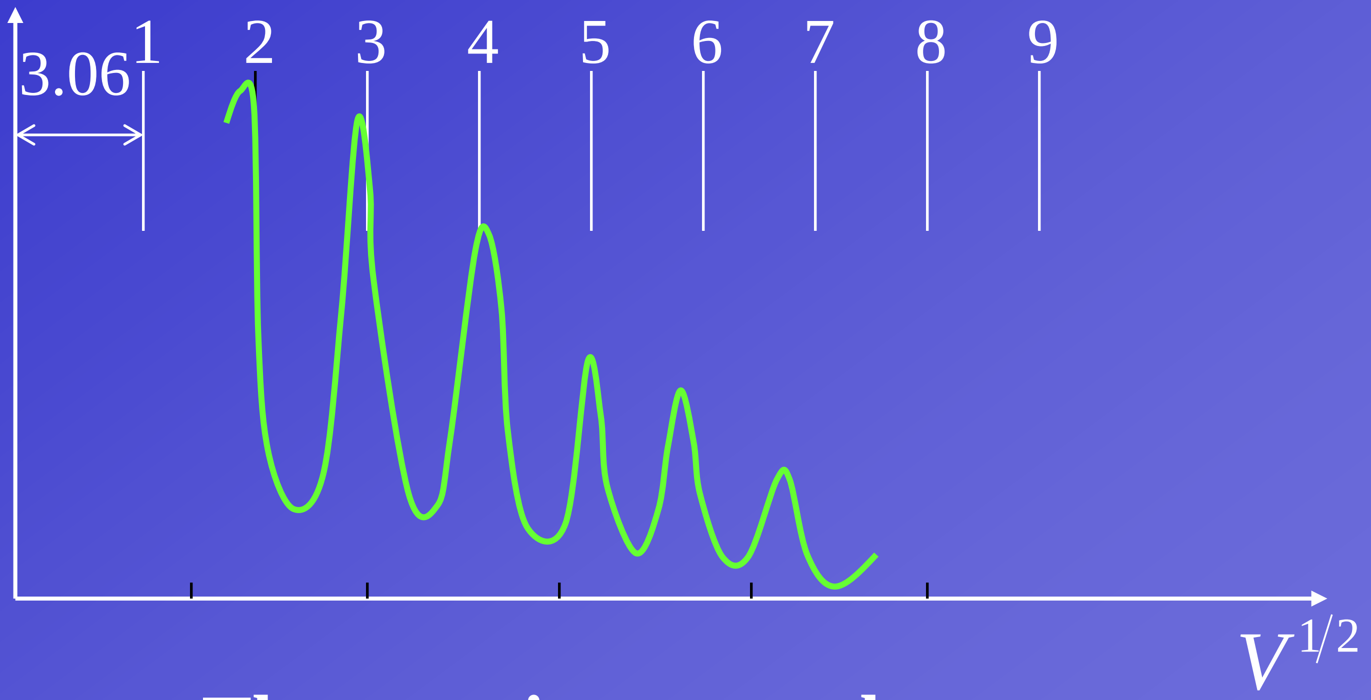
$$\frac{1}{2} m_0 v^2 = eU$$

$$\lambda = \frac{h}{\sqrt{2em_0U}} = \frac{12.3}{\sqrt{U}}(\overset{\circ}{A})$$

For maximum current

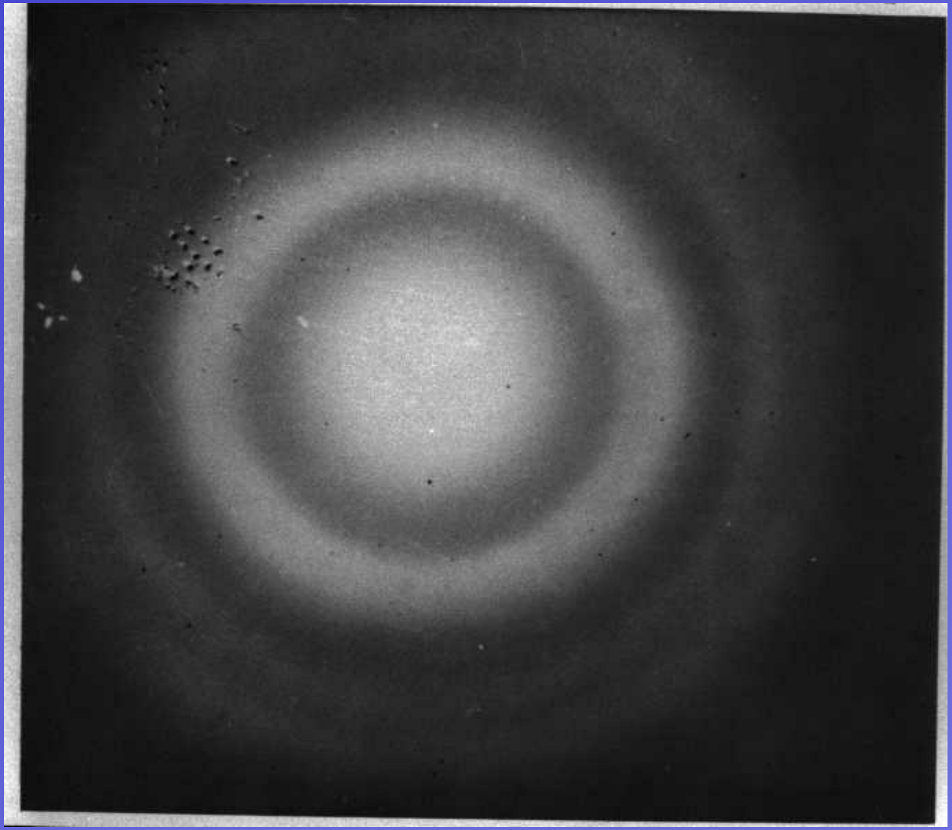
$$2d\sin\phi = k\frac{12.3}{\sqrt{U}} \quad (k = 1, 2, 3, \cdots)$$

$$\sqrt{U} = k\,\frac{12.25}{2d\sin\theta}$$



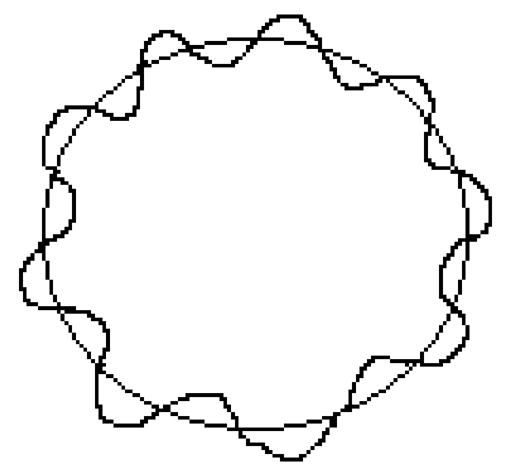
The experiment result

The diffraction of electrons by thin gold foil



G.P.Thomson

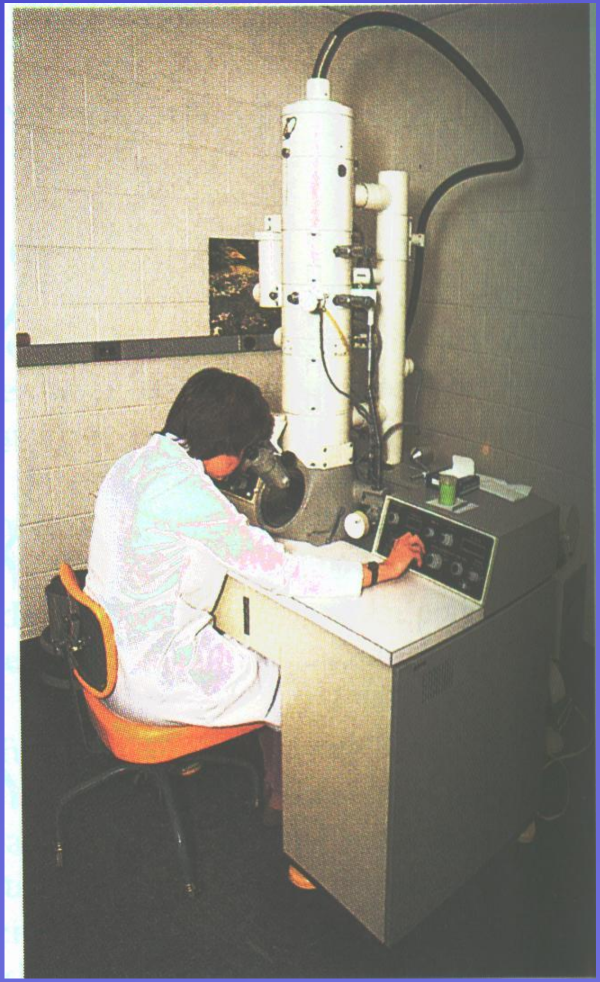
The quantification of angular momentum



$$2\pi r = n\lambda = n\frac{h}{mv}$$

$$\therefore mvr = n\frac{h}{2\pi} = n\hbar$$

Electron microscope



In a electron microscope, electrons are accelerated by potential difference of 100kV, find the de broglie wavelength.

Solution:

Since the kinetic energy is quite large, the effect of relativity must be considered here.

$$E = E_0 + E_k = E_0 + eU$$

$$E^2 = E_0^2 + p^2c^2$$

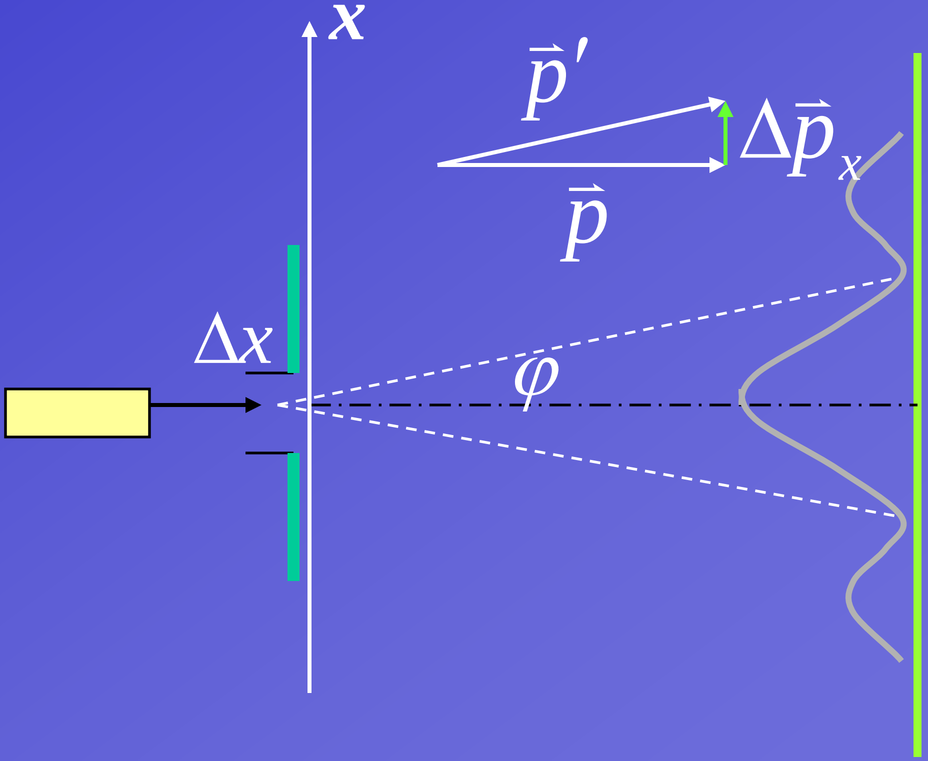
$$\lambda = \frac{h}{p} = 0.0370 \text{ Å}$$

Find the de Borglie wavelength of a bullet with m=0.01kg, speed v=300m/s

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.62\times10^{34}}{0.01\times300} = 2.21\times10^{34}\,m$$

The uncertainty principle（测不准原理）



$$\Delta p_x = \left| \vec{p}' - \vec{p} \right| = p\,tg\varphi \approx \varphi\, p$$

$$\Delta x \varphi = \Delta x \sin \varphi = \pm \lambda$$

$$\frac{\Delta p_x}{p} = \frac{\lambda}{\Delta x}$$

$$\because \quad p = \frac{h}{\lambda} \qquad \Delta x \cdot \Delta p_x = h$$

It is impossible to measure the position and linear momentum of a particle simultaneously.

In strict form

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

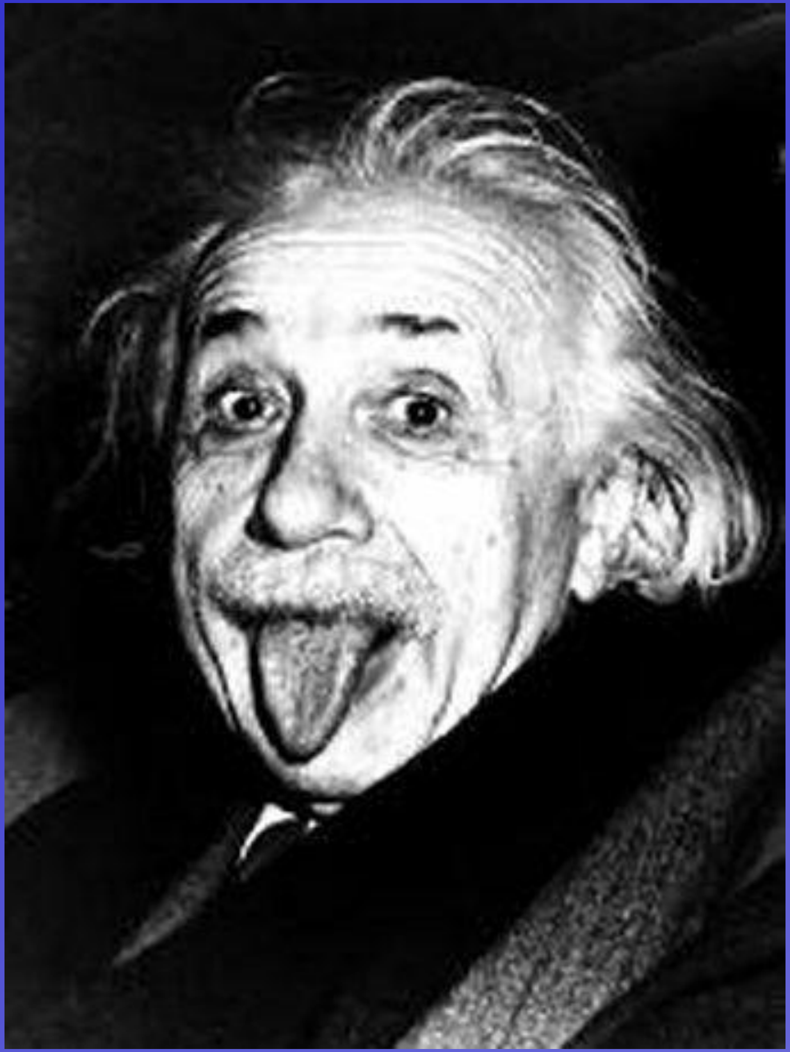
Also the accuracy for measurement of particles energy depends on the time interval needed for the measurement.

$$\Delta E \cdot \Delta t \geq \hbar / 2$$

I would like to put the uncertainty principle in its historical place: When the revolutionary ideas of quantum physics were first coming out,people still try to understand them in term of old fashioned ideas….If you get rid of these old-fashioned ideas and instead use the ideas that I'm explaining in these lectures….There is no need for a uncertainty principle.

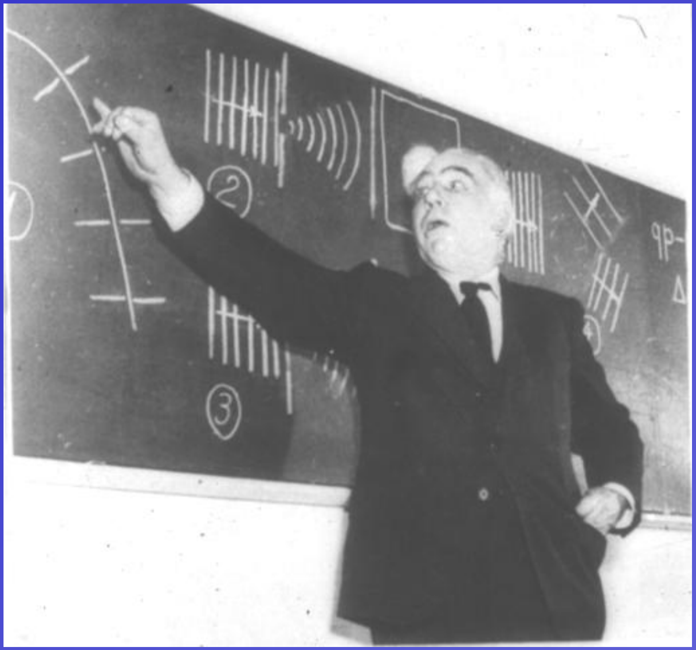
R.P.Feynman

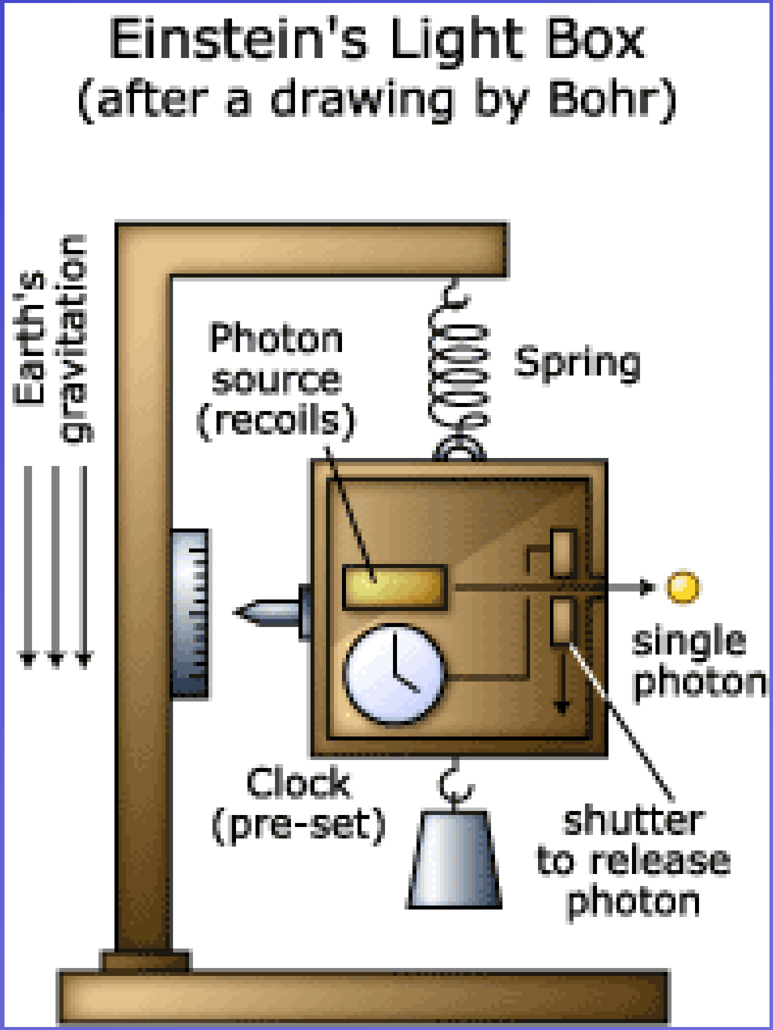
Einstein argues



The Quantum Mechanics is very imposing. But an inner voice tells me that it is still not the final truth. The theory yields much, but it hardly brings us nearer to the secret of the Old One. In any case, I am convinced that He does not play dice.
Albert Einstein

Bohr fought back





Example: Use the position momentum uncertainty relation to estimate the lowest energy of a particle of mass m in a one-dimensional box with width L.

Solution:

Since the particle is somewhere inside the box, so $\Delta x = L$

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{2L}$$

$$E_{\min} = \frac{p_{\min}^2}{2m} = \frac{\hbar^2}{8mL^2}$$